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AUTHOR White, Jacci Wozniak; Norwich, Vicki Howard
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ABSTRACT

Mathematics software can be a great aid in understanding difficult mathematics concepts at all levels. This paper presents nine exercises on calculus concepts by using different software used in mathematics education. Each exercise includes instruction on how to use software in order to highlight a specific concept in mathematics. This paper also presents a technology review by the comparison of ISETL, Derive, Geometer's Sketchpad, TI-82, TI-83, TI-85, TI-86, TI-92 and CBL along with discussing the advantages and disadvantages of each system. The mathematical concepts and software used in this paper include: (1) vertices of a triangle, midpoint formula, median of a side, equation of a line, and slope using the software "The Geometer's Sketchpad"; (2) writing the equation of a line using slope-intercept form and point-slope form and writing equations of parallel and perpendicular lines to a given line and passing through a given point using the software "The Geometer's Sketchpad"; (3) visualizing trigonometric identities for sine, cosine, and tangent functions using the software "The Geometer's Sketchpad"; (4) using equations and graphs to identify families of functions including linear, quadratic, and exponential using the software "Derive"; (5) functions, vertical line test and domain, using the software "Power Point"; (6) Pascal's triangle, binomial expansion, and pattern recognition using the software "Power Point"; (7) graphing factored polynomials of degree n>1 and solving higher degree inequalities by factoring and graphing using the software "Microsoft PowerPoint"; (8) review of quadratic functions and application problems using quadratic functions using the software "Microsoft PowerPoint and Microsoft Excel"; and (9) shifting graphs including horizontal, vertical, and combination graph shifts using the software "Derive". (ASK)

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COMPUTER ACTIVITIES FOR COLLEGE ALGEBRA AND PRECALCULUS

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Jacci Wozniak White
Assistant Professor of Mathematics
Brevard Community College, Melbourne campus
E-mail: WOZNIAK.J@A1.BREVARD.CC.FL.US

Vicki Howard Norwich
Assistant Professor of Mathematics
Brevard Community College, Melbourne campus
E-mail: NORWICH.V@A1.BREVARD.CC.FL.US

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EXERCISE #1

Vertices of a Triangle
Midpoint Formula
Median of a Side
Equation of a Line
Slope
Software: The Geometer's Sketchpad

EXERCISE #1

Vertices of a Triangle, Midpoint Formula, Median of a Side, Equation of a Line, and Slope

Date: _____ **Group Members:**

_____

Software Required:
The Geometer's Sketchpad

_____

_____

_____

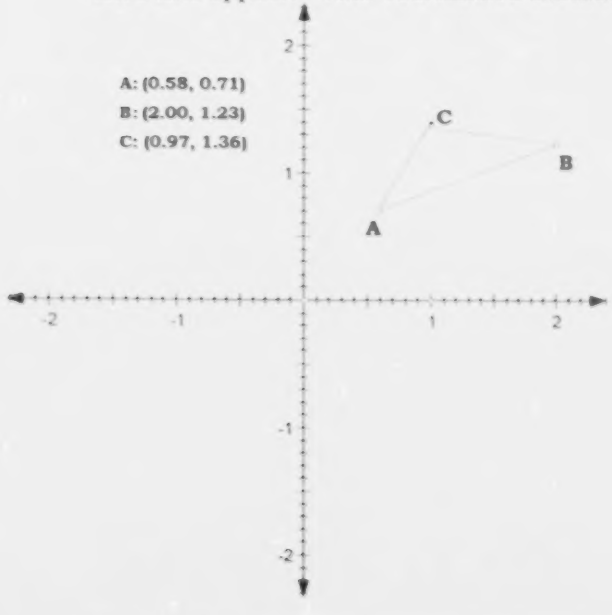
The student will learn how to use Sketchpad's coordinate system to measure coordinates and plot points. The student will also write the equation of a line and check that the coordinates of a given point satisfy the equation. The student will also need to review the midpoint formula before beginning this exercise.

PART I

Vertices of a Triangle, Midpoint Formula, Median of a Side:

1. Start with a new sketch. Draw a triangle in the first quadrant and find the coordinates of its vertices.
   - Use the segment tool to draw the triangle in the first quadrant.
   - Select the three vertices and choose **Coordinates** from the **Measure** menu.

   Your sketch should look similar to the one below. The coordinates and a pair of axes also appear. Your coordinates should not match the example below.



   - Try dragging the vertices of the triangle and notice how the coordinates change.

2. Write down the coordinates of the points of the triangle that you have currently on the screen.

Point **A** (_____, _____) Point **B** (_____, _____) Point **C** (_____, _____)

Use your calculator to compute the mean of the x-coordinates of the three points. Show all of your work in the next few steps:

$$\frac{x_A + x_B + x_C}{3} = \frac{\quad + \quad + \quad}{3}$$

$$= \frac{\quad}{3}$$

= _____ (final answer)

Use your calculator to compute the mean of the y-coordinates of the three points. Show all of your work in the next few steps:

$$\frac{y_A + y_B + y_C}{3} = \frac{\quad + \quad + \quad}{3}$$

$$= \frac{\quad}{3}$$

= _____ (final answer)

3. Verify your answer from step #2 by using Sketchpad:
   - Select the three coordinates by clicking on the summary of what the coordinates are for the vertices of the triangle. (Use shift-select to select all three points)
   - Choose **Calculate** from the **Measure** menu.
   - Start by pressing the **left open parenthesis** key. You want to make sure that the sum of the x-values is what you get in the numerator of the fraction.
   - Hold down the mouse pointer on the **Values** pop-up menu. Move the pointer to Point **A**, then move to the right to choose x in the cascading menu. The calculator display shows x_A .
   - Press the **+** key (plus) on the calculator keypad.
   - Next choose x_B on the **Values** pop-up menu.
   - Press **+** key (plus) on the calculator keypad once again.
   - Choose x_C on the **Values** pop-up menu.
   - Press the **right open parenthesis**.
   - Press **/** and then **3** on the calculator keypad.
   - Press the **OK** button. The calculation should appear in the sketch window.

$$\frac{x_A + x_B + x_C}{3} = \text{\_\_\_\_\_}$$

4. Now repeat the above calculation for the mean of the y-coordinates. After following the steps, the calculation you needed should appear in the sketch window.

$$\frac{y_A + y_B + y_C}{3} = \text{\_\_\_\_\_}$$

5. Now you are going to plot another point by using the mean of the x-coordinates and mean of the y-coordinates.

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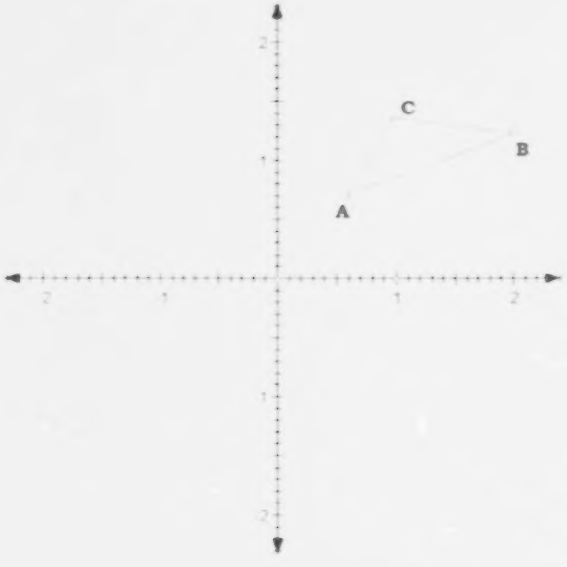
- Select on your screen the calculation that represents the mean of the x-coordinates, then use the Shift-select to select the calculation that represents the mean of the y-coordinates.
- Choose **Plot** as **(x,y)** from the **Graph** menu.



A new point appears in the middle of the triangle. Drag the vertices of the triangle and write below how the new point seems to respond:

_____

_____

6. The plotted point represents the centroid of the triangle. You should check this guess by drawing some of the median lines of the triangle.

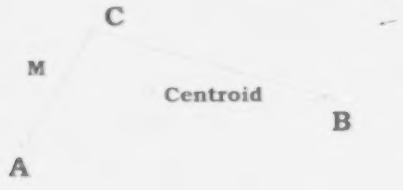
- Consider segment AB.
- Use your calculator to find the midpoint of segment AB.

$$\text{midpoint of segment AB} = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

$$= (\quad , \quad)$$

- Next, select the segment AB on your screen.
- Choose **Midpoint** from the **Construct** menu.
- Select point C and the midpoint of segment AB.
- Choose **Line** on your **Toolbar** (not segment - hold down the segment button until you see other options on the Toolbar) then select **Line** from the **Construct** menu.
- Notice that the median line appears to go through the plotted point no matter how you drag the triangle.
- Next, select the segment **BC** on your screen.
- Choose **Midpoint** from the **Construct** menu.

- Select point **A** and the midpoint of segment **BC**.
- Choose **Segment** from the **Construct** menu.
- Notice that the median line still appears to go through the plotted point.
- Label the plotted point **Centroid**
- Label the midpoint of the line segment AC by using the label **M** (Select the **Rename Label** in the **Display** menu and type in **M**



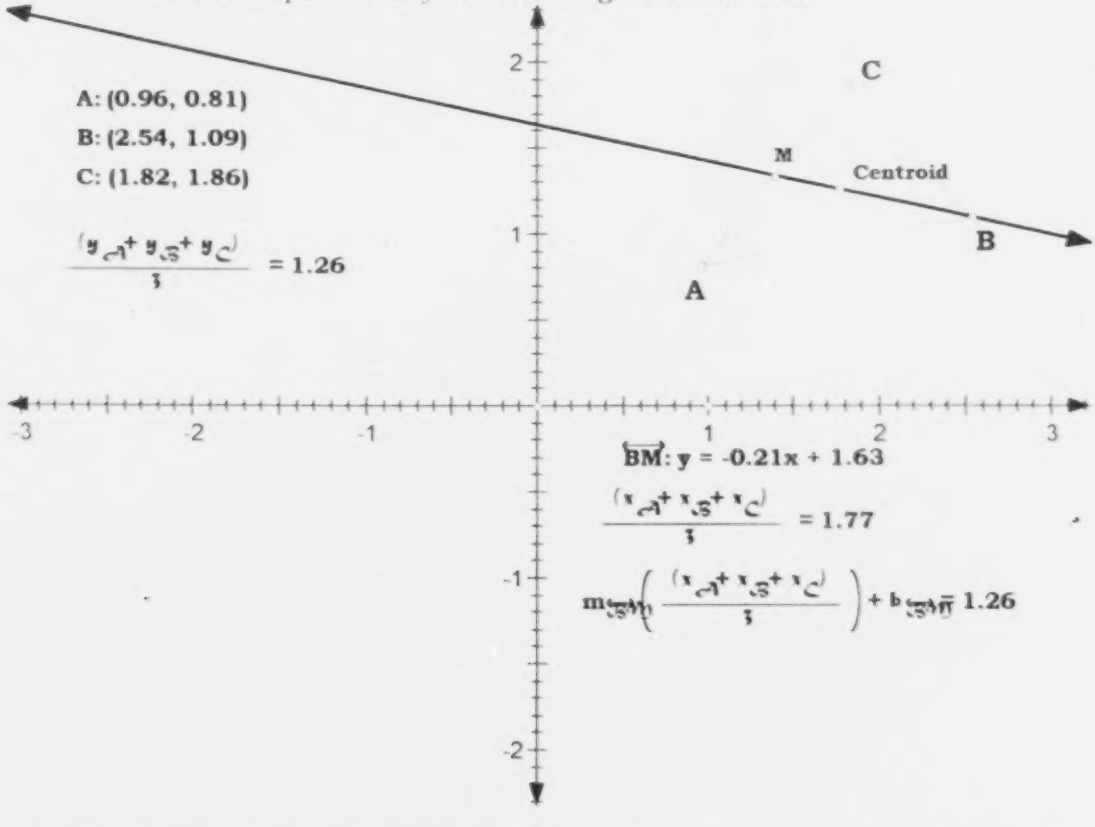
PART II

Equation of a Line and Slope

1. Next you will find the equation for a median line for the triangle you left off with in Part I. You will determine whether the coordinates of the centroid satisfy the equation.

   - Select a median line that you contructed from Part I.
   - Choose **Equation** from the **Measure** menu. (Notice that if you drag the vertices of the triangle, the equation of the median will change dynamically.
   - Select the mean of the x-coordinates and the equation of the line.
   - Choose **Calculate** in the **Measure** menu.
   - In the **Value** pop-up menu, choose the slope of the line.
   - Press * key (asterisk) on the keypad.
   - In the Value menu, choose the mean of the x-coordinates.
   - Press + key (plus) on the keypad.
   - In the Value pop-up menu, choose the intercept of the line.
   - Press OK.

   Here is an example of what your screen might look like now:



The result of substituting a particular x into the equation of the line appears. Compare it with the mean of the y-coordinates. Drag the vertices of the triangle to check that equality of the two computations continues to hold true. You could use your calculator and prove that the medians of a triangle all go through a single point.

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REVIEW QUESTIONS:

1. How do you get the coordinates of a point and the equation of a line?

2. How do you get the x-coordinate of a point in a calculation?

3. Given the computed coordinates of a point, how would you plot the point in a a coordinate system?

Vicki Howard Norwich
Assistant Professor of Mathematics, Brevard Community College, Melbourne, Florida
Jacci Wozniak White
Assistant Professor of Mathematics, Brevard Community College, Melbourne, Florida

EXERCISE #2

Writing the Equation of a Line using Slope-Intercept Form and Point-Slope Form

Writing equations of Parallel and Perpendicular lines to a given line and passing through a given point.

Software: The Geometer's Sketchpad

EXERCISE #2

Writing the Equation of a Line using Slope-Intercept Form and Point-Slope Form

and

Parallel Lines and Perpendicular Lines to a Fixed Line and a Given Point

Date: _____ **Group Members:**

_____

Software Required:
The Geometer's Sketchpad

_____

_____

_____

The student will use Sketchpad's coordinate system to plot two points and then write the equation of the line passing through the two points. The student will also write the equation of a line using the slope-intercept form and the point-slope form of a line. The student will also need to review the slope-intercept form and the point-slope form of a line as well as concepts related to parallel and perpendicular lines to a given line and a point.

Plotting Two Given Points

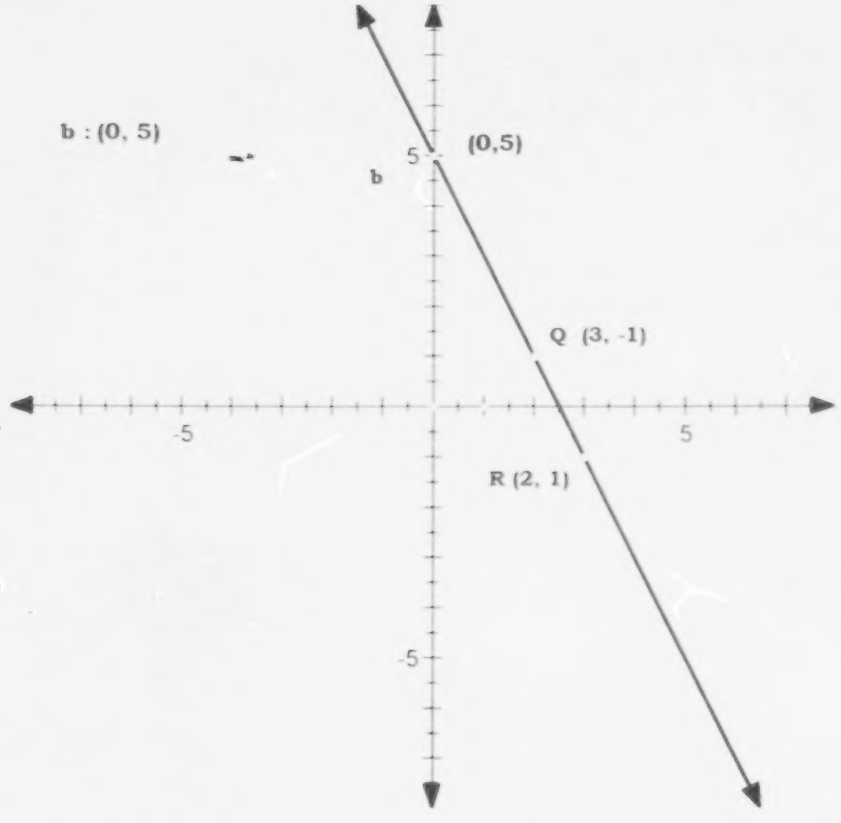
1. Start with a new sketch. Plot the points (3, -1) and (2, 1) on a coordinate system.

   - Select **Create Axes** from the **Graph** menu.
   - Select the point at (1,0) and drag it left to change the scale on your axes.
   - Select **Plot Points** from the **Graph** menu.
   - Type in the window that appears the two points listed above: (3, -1) and (2, 1) .
   - Click on the **Line** tool in the **Toolbar** (hold down the Segment tool until you see the option of drawing a line)
   - Select **Line** in the **Construct** menu.
   - Label each point by selecting **Relabel objects** in the **Display** menu.
     Type in Q (3, -1) for the appropriate point and R (2, 1) for the coordinates of the other point.
   - Select the point where the line intersects the y-axis. Select **Coordinates** from the **Measure** menu. How will you be sure that it is the y-intercept?

_____

_____

Your sketch should look similar to the one on the next page.



2. Write down the coordinates of the points and use the Point-Slope Form to write the equation of the line shown above. **SHOW ALL OF YOUR WORK.**

a) Use the slope formula you have learned to find the slope of the line.

Slope = _____

b) Use one of the points Q or R that you plotted and the slope of the line to write the equation of the line using the Point-Slope Form of a line.

Write your equation in standard form: Ax + By = C
Write it on the line below:

Equation of the line = _____

3. Write down the y-intercept of the line and also the slope you found in step #2 to write the equation of the line using the slope-intercept form of a line.

SHOW ALL OF YOUR WORK.

a) Use the slope you found in step #2: **slope**: _____

b) Write down the y-intercept of the line that you have found in Sketchpad.

y-intercept: _____

c) Use the slope-intercept form of a line to write the equation of the line passing through Q and R.

Slope-Intercept Form of the line = _____

4. Verify your answer from step #2 by using Sketchpad:

- Select the line passing through the two points (3, -1) and (2, 1).
- Select **Slope** from the **Measure** menu. Write the answer below:

slope: _____

Write your slope answer from step #2. _____

How do they compare?

- Select **Equation Form** from the **Graph** menu.
- Select **Standard form** from the options.
- Select **Equation** from the **Measure** menu. Write the answer below:

standard form of the line:

_____

- Write the standard form of the line you wrote in Step #2:

_____

- How do the two answers compare?

5. Verify your answer from step #3 by using Sketchpad:

- Once again, select the line passing through the two points (3, -1) and (2, 1).
- Select **Slope** from the **Measure** menu. Write the answer below:

slope: _____

- Select **Equation Form** from the **Graph** menu.
- Select **Slope-Intercept** form from the options.
- Select **Equation** from the **Measure** menu. Write the answer below:

slope-intercept form of the line:

_____

- Write your answer from step #3.

slope-intercept form of the line:

_____

- How do the two answers compare?

PART II

Equation of a Line Parallel to a given line and a given point.

1. Next you will find the equation for a line parallel to a given line and a given point.

   - Select the line QR that you have on the screen.
   - Plot the point (-1,4).
   - Select the point and the line.
   - Select **Parallel Line** from the **Construct** menu.
   - Find the slope of this new line by selecting Slope in the Measure menu.

     Slope = _____

   - How does it compare to the slope of the line passing through Q and R?

2. Use the point you plotted and the slope of the new line to find the equation of the new line. **SHOW ALL YOUR WORK.**

   Write the equation of the line in slope-intercept form._____

3. Verify your answer from step #2.
   - Select the new line.
   - Select **Equation Form** in the **Graph** menu. Choose slope-intercept form.
   - Select **Equation** in the **Measure** menu.

   Write the equation of the line in slope-intercept form that you obtained from Sketchpad:

   _____

   How do the two answers compare?

4. Next you will find the equation for a line perpendicular to a given line and a given point.

   - Select the line QR that you have on the screen.
   - Plot the point (2,-3)
   - Select the point and the line.
   - Select **Perpendicular Line** from the **Construct** menu.
   - Find the slope of this new line by selecting Slope in the Measure menu.

     Slope = _____

   - How does it compare to the slope of the line passing through Q and R?

5. Use the point you plotted and the slope of the new perpendicular line to find the equation of the new line. **SHOW ALL YOUR WORK.**

Write the equation of the line in slope-intercept form. _____

6. Verify your answer from step #4.
   - Select the new line.
   - Select **Equation Form** in the **Graph** menu. Choose slope-intercept form.
   - Select **Equation** in the **Measure** menu.

Write the equation of the line in slope-intercept form that you obtained from Sketchpad:

_____

How do the two answers compare?

Review Questions concerning Sketchpad:

1. How do you plot a given point?

2. How do you get the slope of a line?

3. How do you get the equation of a line to appear on the screen?

4. Given a fixed point and a line, how do you construct a line perpendicular to the given line.

Vicki Howard Norwich
Assistant Professor of Mathematics, Brevard Community College, Melbourne, Florida
Jacci Wozniak White
Assistant Professor of Mathematics, Brevard Community College, Melbourne, Florida

EXERCISE #3

Visualizing Trigonometric Identities for Sine, Cosine, and Tangent Functions

Software: The Geometer's Sketchpad

EXERCISE #3

Constructing Trigonometric Identities for Sine, Cosine, and Tangent

Date: _____

Group Members:

_____

_____

_____

_____

Software Required:
The Geometer's Sketchpad

The student will learn how to construct a perpendicular line, measure angles, and compute ratios on the Sketchpad. The student will use these features in order to construct a right triangle and measure the ratio of the lengths of the sides in order to visualize trigonometric identities for the sine, cosine, and tangent functions.

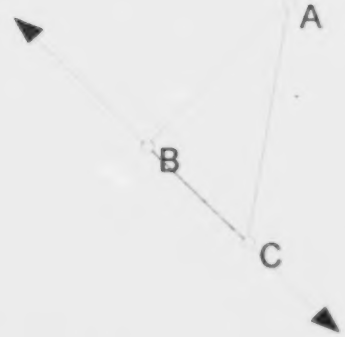
PART 1

Measurement of Angles and Sides of a Right Triangle to Explore the Sine Identity as a Ratio of Opposite over Hypotenuse:

1. Open the file "Sine1" in The Geometer's Sketchpad. Your screen should contain the following sketch.

2. Use Sketchpad to measure the following:
   - angle BAC
   - the length of side AB
   - the length of side AC
   - the length of side BC

Your sketch should now contain all of the following data



$m\angle BAC = 35°$

$m\overline{BA} = 0.86$ inches

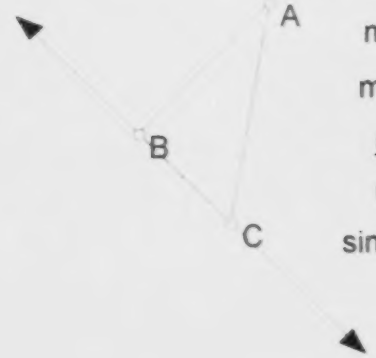
$m\overline{CA} = 1.05$ inches

$m\overline{BC} = 0.60$ inches

3. Now use Sketchpad to Calculate the following

- The ratio of the length of the opposite side to angle BAC to the length of the hypotenuse
- The sine of angle BAC

Your sketch should now contain all of the following data



$m\angle BAC = 35°$

$m\overline{BA} = 0.86$ inches

$m\overline{CA} = 1.05$ inches

$m\overline{BC} = 0.60$ inches

$\frac{m\overline{BC}}{m\overline{CA}} = 0.57$

$\sin(m\angle BAC) = 0.57$

4. Now grab point A and begin to move it to answer the following questions.

a. What do you notice about the length of the sides of the triangle as you move point A?

b. What do you notice about the measurement of Angle BAC as you move point A?

c. What do you notice about the ratio of length of the opposite side to the length of the hypotenuse as you move angle A?

d. Does the ratio of the lengths of the given sides change as the length of the sides change?

e. What do you notice about the sine of angle BAC as you move point ?

f. What do you notice about the sine of angle BAC and the ratio of the length of the opposite side to the length of the hypotenuse for angle BAC?

5. What can you tell me about the sine of an angle?

PART II

Exploration of the Cosine and Tangent Functions

1. Delete the ratio and sine off of your sketch for Part I. (or open "sine2" in the Sketchpad if you do not have a sketch from Part I). Your sketch should look like the following:

$m\angle BAC = 35°$

$m\overline{BA} = 0.86$ inches

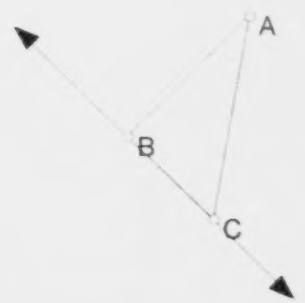
$m\overline{CA} = 1.05$ inches

$m\overline{BC} = 0.60$ inches

2. Now use Sketchpad to Calculate the following:

- The ratio of the length of the adjacent side to angle BAC to the length of the hypotenuse
- The cosine of angle BAC

Your sketch should now contain all of the following data



$m\angle BAC = 35°$

$m\overline{BA} = 0.86$ inches

$m\overline{CA} = 1.05$ inches

$m\overline{BC} = 0.60$ inches

$\frac{m\overline{BA}}{m\overline{CA}} = 0.82$

$\cos(m\angle BAC) = 0.82$

3. Now grab point A and begin to move it to answer the following questions.

a. What do you notice about the length of the sides of the triangle as you move point A?

b. What do you notice about the measurement of Angle BAC as you move point A?

c. What do you notice about the ratio of length of the adjacent side to the length of the hypotenuse as you move angle A?

d. Does the ratio of the lengths of the given sides change as the length of the sides change?

e. What do you notice about the sine of angle BAC as you move point A?

f. What do you notice about the cosine of angle BAC and the ratio of the length of the adjacent side to the length of the hypotenuse for angle BAC?

4. What can you tell me about the cosine of an angle?

5. Repeat the above set of exercises for the ratio of the length of the side opposite angle BAC to the length of the side adjacent to angle BAC. This time Calculate the tangent of angle BAC rather than the cosine.

6. What can you tell me about the tangent of an angle?

JUST FOR FUN What do you think **SOH - CAH - TOA** stands for?

EXERCISE #4

Using Equations and Graphs to Identify Families of Functions Including: Linear, Quadratic, and Exponential

Software: Derive

EXERCISE #4

Using Equations and Graphs to Identify Families of Functions Including: Linear, Quadratic, and Exponential

Date: _____ **Group Members:**

_____

Software Required:
Derive

_____

_____

_____

The student will develop the concept of qualities that are similar within each family of functions. First, the student will classify a set of equations by similarities they choose. Next, Derive will be used to graph each equation so that the graphs can be classified into groups. Finally, the student will match each equation to its graph and classify the pairs into similar groups.

PART 1

Linear, Quadratic, and Exponential Equations and Graphs

1. Linear, Quadratic, and Exponential Equations and Graphs

$f(x) = 2x^2 - 3x + 1$ $g(x) = 5 - 9x$

$s(t) = -5t^2 + 7t$ $h(x) = -2x^2 - 5$ $f(x) = 3^x$

$f(x) = 6^{x+1} - 7$ $f(x) = 1 - 2^x$ $h(t) = (t+1)(t-1)$

$g(t) = -9t + 5$ $f(x) = 6x + 8$ $g(x) = (-6)^{x-3}$

$h(t) = 5t^2 - 7t - 1$ $s(t) = 4^t - 2$

How did you choose the three groups you use?

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2. Open Derive and sketch each of the prior functions in the space provided.

3. Classify each of your graphs into 3 groups in the space provided.

How did you choose the three groups?

4. Do the three groups of equations match the three groups of graphs? If not, put each equation with its graph and break the pairs into three groups where the equations and the graphs all have qualities in common within each group.

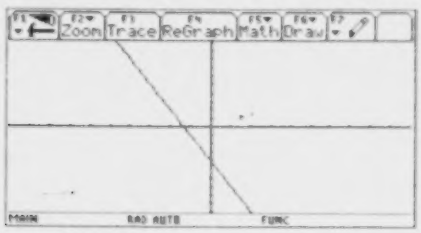
Share your results with the class and determine three names for families of functions that fit the given functions.

PART II

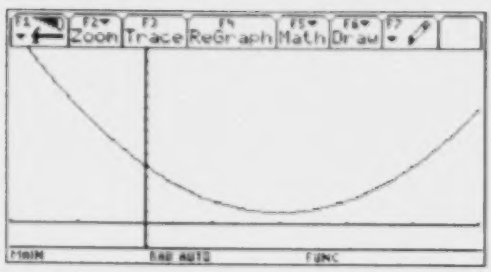
Practicing Classification of Functions by Equation and Graph.

1. Classify each of the following equations and/or graphs as either:
   - Linear
   - Quadratic
   - Exponential

Give a reason for each of your answers.



$f(x) = x^2 + 3x$ $\quad$ $h(t) = (t+1)(t+5)$ $\quad$ $s(t) = 5 + 9^t$



$f(x) = 1$ $\quad$ $f(x) = 2^{3x-5}$

$h(x) = 9 - x^2$ $\quad$ $g(x) = -3x + 1$

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EXERCISE #5

Functions:
Vertical Line Test
And Domain
Software: Power Point

EXERCISE #5

Functions: Vertical Line Test and Domain

Date: _____ **Group Members:**

_____

Software Required:
Power Point

_____

_____

_____

The student will learn how to identify a function by looking at the graph of equations. The student will also practice identifying the domain of various functions.

PART 1

Functions: Vertical Line Test

1. Open the PowerPoint file titled "functions" and read slides #1-3 to answer the following questions.

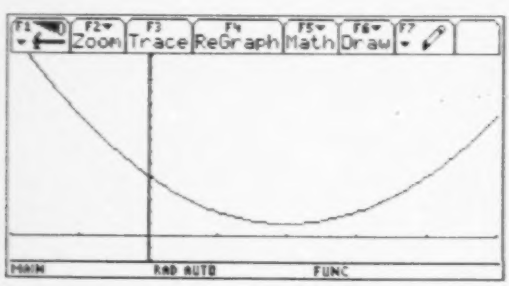
- What is a function?

- How can you identify a function from the graph of an equation?

2. Follow the directions for slides #4-15 and answer the following questions.

- Is this the graph of a function?

Why or Why not?



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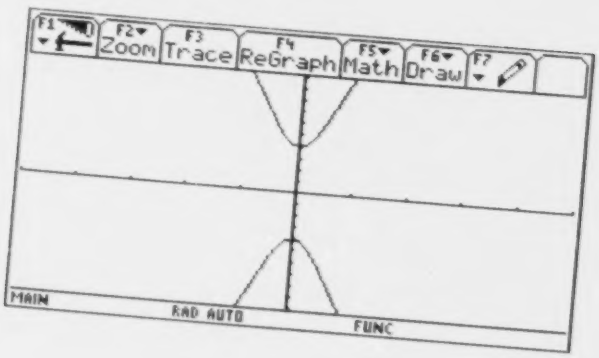
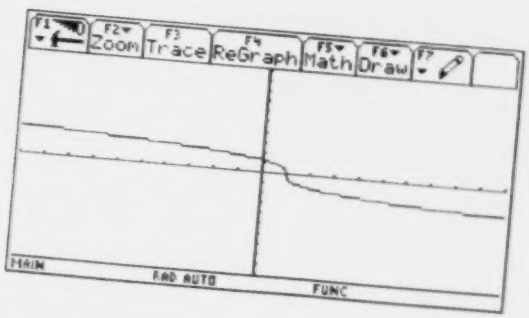
3. Is this the graph of a function?

Why or Why not?



4. Is this the graph of a function?

Why or why not?



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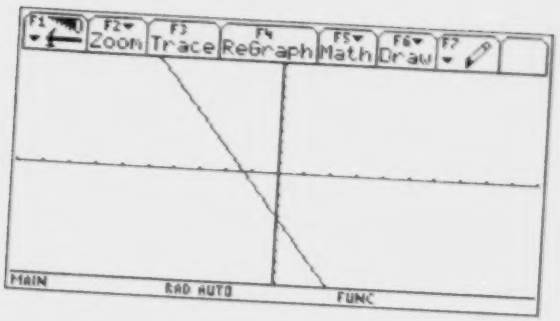
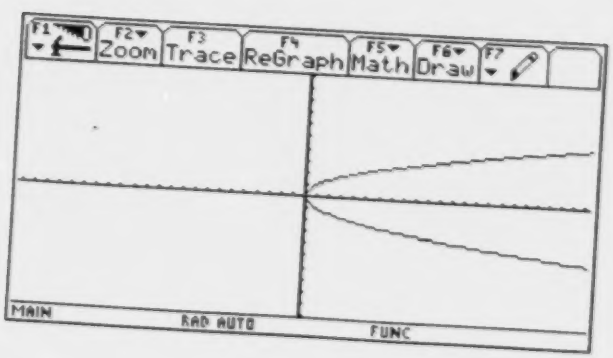
5. Is this the graph of a function?

Why or why not?



6. Is this the graph of a function?

Why or why not?



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PART II

Domain of a Function

1. Continue exploring PowerPoint file "functions", looking at slides #16-32 to answer the following questions.

   - What is one restriction on the domain of a function?

   - What is the domain of this function?

$$f(x) = \frac{x^2 + 1}{x - 2}$$

   - What is the domain of this function?

$$f(x) = \frac{x^2 + 2x - 1}{x(x + 3)(x - 4)}$$

2. What is another restriction on the domain of a function?

   - What is the domain of this function?

$$g(x) = \sqrt{2x + 3}$$

- What is the domain of the function f(x) = x-2? f(x) = 4x-7? f(x)=(x+5)(x-2)? Why?

- What is the domain of this function?

$$s(t) = \frac{t+4}{t^2-9}$$

- What is the domain of this function?

$$f(x) = x^3 - 2x + 9$$

- What is the domain of this function?

$$g(x) = 2 - \sqrt{3x}$$

COLLEGE ALGEBRA

Sponsored in Part by ACTE and NSF

By
Vicki Norwich and
Jacci White

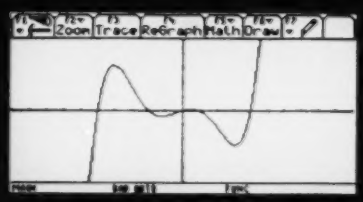
Activity 1

FUNCTIONS

FUNCTIONS

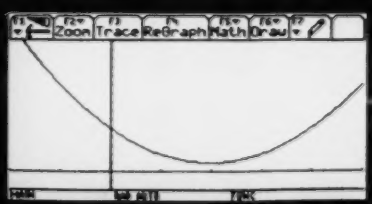
- A function is a rule that assigns a single output to each input
- Definition: A relation that assigns to each member of its domain exactly one member, its range
- Vertical line test: If it is possible for a vertical line to intersect a graph more than once, the graph is not the graph of a function.

This example is a function because no matter where you draw a vertical line it crosses the graph no more than 1 time.



This example is not a function because if you draw a vertical line anywhere near the middle of the graph, it will cross more than one time.

Practice: Is this the graph of a function?



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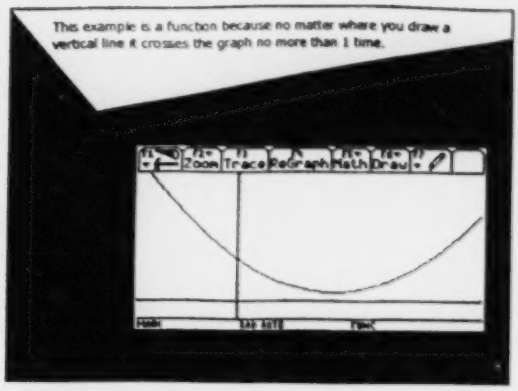
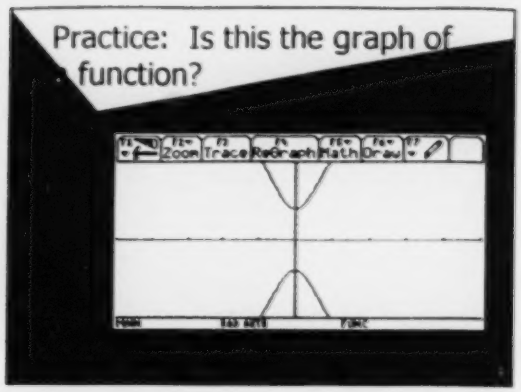
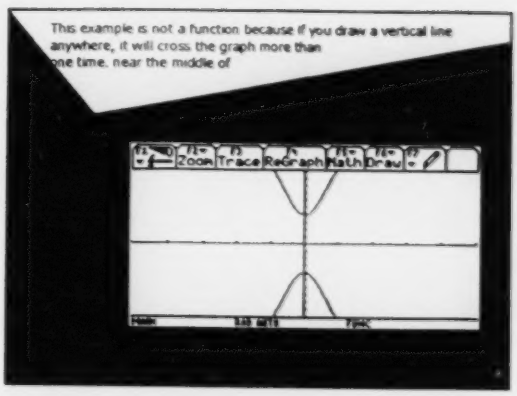
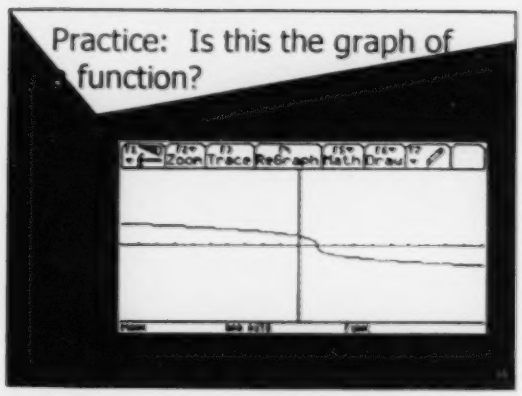
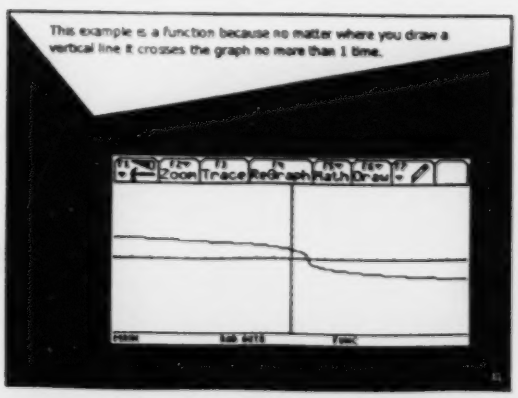
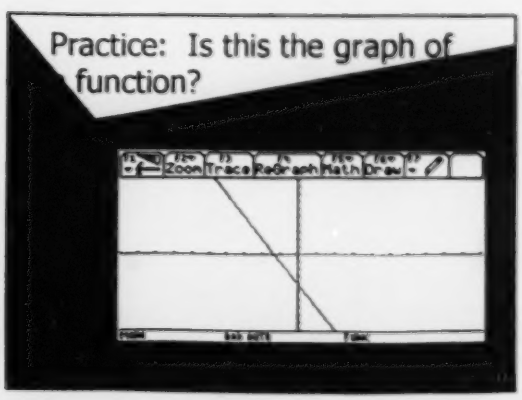

This example is a function because no matter where you draw a vertical line it crosses the graph no more than 1 time.
Zoom
Trace
ReGraph
Math
Draw




This example is not a function because if you draw a vertical line anywhere, it will cross the graph more than one time. near the middle of
Zoom
Trace
ReGraph
Math
Draw




This example is a function because no matter where you draw a vertical line it crosses the graph no more than 1 time.
Zoom
Trace
ReGraph
Math
Draw



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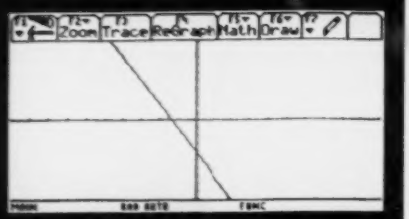
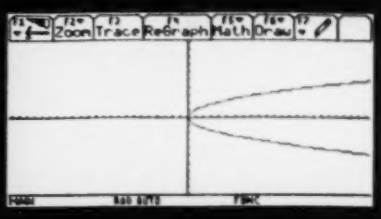
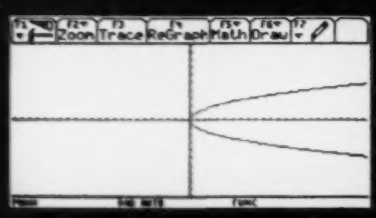
This example is a function because no matter where you draw a vertical line it crosses the graph no more than 1 time.



Practice: Is this the graph of a function?



This example is not a function because if you draw a vertical line anywhere on the right side of the graph, it will cross more than one time.



DOMAIN

- The Denominator cannot equal zero. So the domain is made up of all real numbers that will not make the denominator equal to zero

Examples

- The domain of this function is the set of all real numbers not equal to 3

$$f(x) = \frac{x+7}{x-3}$$

What is the domain of f(x)?

$$f(x) = \frac{x^2+1}{x-2}$$

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Answer

Another way to write the answer is:

$\{x \in \mathbb{R} : x \neq 2\}$

Another example of a function and the domain.

function

domain

Practice: What is the domain of the following function?

Your answers should be:

- The set of all real numbers not equal to 0, 4, and

$\{x \in \mathbb{R} : x \neq 0, 4, -3\}$

Domain

- An even radicand must be greater than or equal to zero. In other words, an even radicand can never be negative.

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Examples

- The following function is a square root function. Because square root is even, the part of the function under the square root sign must be greater than or equal to zero.

$$f(x) = \sqrt{x-1}$$

Solution

- To find the

Practice: What is the domain of the following function?

$$g(x) = \sqrt{2x+3}$$

Your answers should be:

All rules for domains must be used whenever they apply.

- What is the
- What is the
- What is the

Solution: All real numbers

The solution is all

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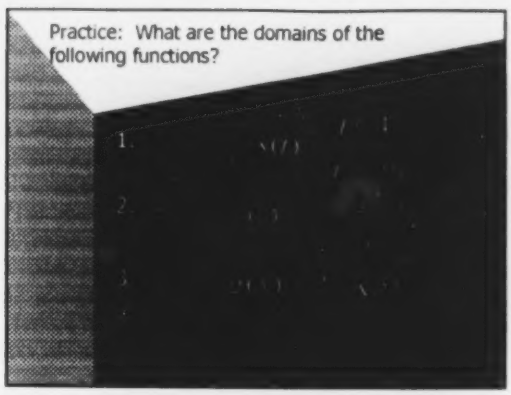

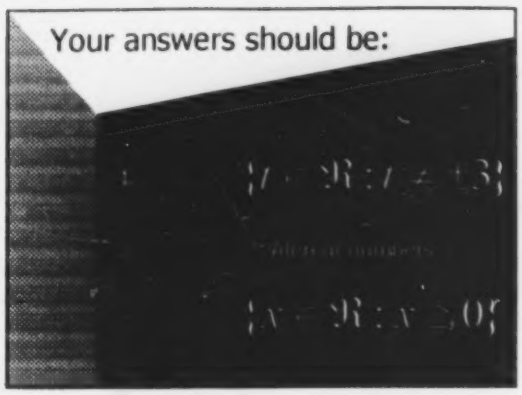

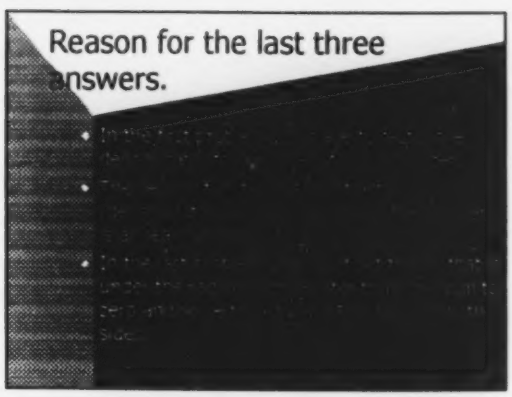

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EXERCISE #6

Pascal's Triangle, Binomial Expansion, and Pattern Recognition

Software: Power Point

EXERCISE #6

Pascal's Triangle, Binomial Expansion, and Pattern Recognition

Date: _____

Group Members:

_____

_____

_____

_____

Software Required:
Power Point

The student will explore the relationship between Pascal's triangle and binomial expansion. These ideas will be used to recognize patterns that lead to infinite sequences and series.

PART 1

Pascal's Triangle

1. Open the PowerPoint file titled "Pascal" and read slides #1-9 to answer the following questions.

   - What is Pascal's triangle?

   - How can Pascal's triangle be used for expanding binomials?

   - What is another method for expanding binomials?

2. Expand the following binomials.

$f(x) = (x+1)^0$

$f(x) = (x+2)^2$

$f(x) = (x + y)^3$

$f(x) = (y - 1)^3$

$f(x) = (2y + 2)^3$

PART II

Pattern Recognition and Area of a Triangle

This activity explores some of the number patterns found in the Sierpinski triangle. Conclude the Power Point file "Pascal".

Directions: The first four stages of the construction of the Sierpinski triangle are shown below. In subsequent stages, the subdivision continues into smaller and smaller triangles. Use these figures to explore number patterns that emerge as the Sierpinski triangle is developed through successive iterations.

1. Count the number of shaded triangles at each stage 0 through 4.

2. Extend the pattern to predict the number of triangles at stage 5. What constant multiplier can be used to go from one stage to the next?

3. Generalize to find the number of triangles for level n. As n becomes large without bound, what happens to the number of triangles?

AREA OF TRIANGLES

4. Let the area at stage 0 be 1. Find the total shaded areas at stage 1 through 4.

5. Extend the pattern to predict the total area at stage 5. What constant multiplier can be used to go from one stage to the next?

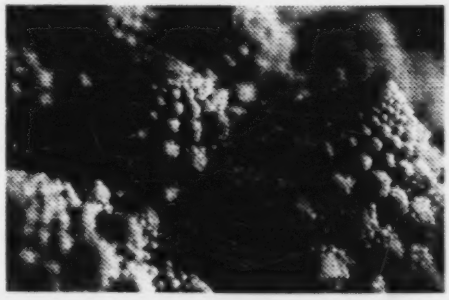
6. Generalize to find the total area at stage n. As n becomes large without bound, what happens to the shaded area?

Pascal's Triangle and Pattern Recognition

By
Vicki Norwich and Jacci White

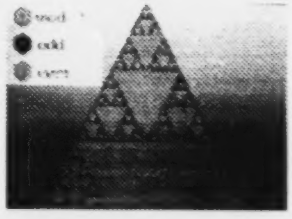
Patterns and Fractals

In a certain sense, an acorn is the whole of the tree that grows from it. The essential features of the tree derive from the code carried in the cells of the acorn. However, some plants exhibit dependency upon their parts in an even more graphic manner. For example, a single stalk of broccoli will be a replication of the entire bunch while a single florette contains the same repeating pattern.



This repeating pattern can be found in many places such as a seashell.

PASCAL'S TRIANGLE



Binomial Expansion

Pascal's triangle is commonly seen as a triangular array of numerical coefficients in the binomial expansion

where the exponent increases through the whole numbers from 0 to n. This triangular array of numbers offers a rich setting for studying both numerical and geometric patterns.

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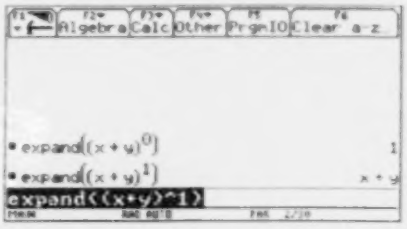
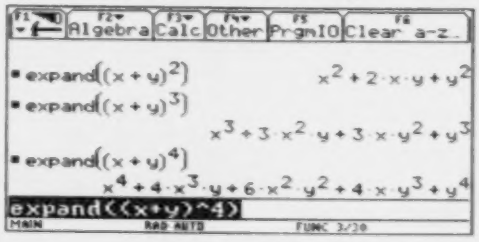
A simple iterative algorithm for generating an entry in the numerical array of numbers in Pascal's triangle is to add the two numbers in the level above it. Unfortunately, the numbers imbedded deeply within the triangle are very large, and this ultimately makes the numerical iteration process increasingly laborious.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
/ \ /\ /\ / \ / \
1 5 10 10 5 1

Here are the expansions from rows 1 and 2 on Pascal's triangle.



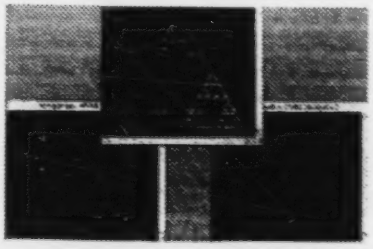
Here are the expansions from rows 3-5 on Pascal's triangle.


F1 F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clear a-z
expand((x+y)^2) x^2+2·x·y+y^2
expand((x+y)^3) x^3+3·x^2·y+3·x·y^2+y^3
expand((x+y)^4) x^4+4·x^3·y+6·x^2·y^2+4·x·y^3+y^4
expand((x+y)^4)
MAIN RAD AUTO FUNC 3/30

Number Patterns and Variations

- Many number patterns can be explored using the Sierpinski triangle.
- The Sierpinski triangle is created through a pattern that connections the midpoints of each side of a triangle to divide each triangle into 4 equivalent triangles. This process is repeated within each new triangle.

SIERPINSKI TRIANGLE



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EXERCISE #7

Graphing Factored Polynomials of Degree n > 1

and

Solving Higher Degree Inequalities By Factoring and Graphing

Software: Microsoft PowerPoint

EXERCISE #7

Graphing Factored Polynomials of Degree n > 1 and Solving Factored Inequalities in One Variable using Graphing Techniques learned in Part I.

Date: _____ **Group Members:**

_____

Software Required:
PowerPoint

_____

_____

_____

The student will learn how to find real zeros of a factored polynomial function and their multiplicities. The student will also learn to sketch factored polynomials and then use the graphing techniques to solve factored inequalities in Part II.

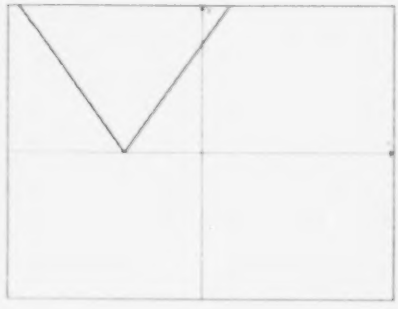
PART I

Graphing Factored Polynomials:

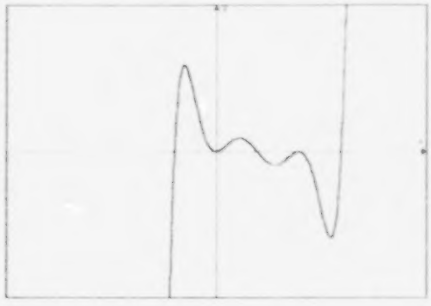
1. Open the PowerPoint file titled "POLYN" and read slides #1-3 to answer the following question.

   Which graphs below are "continuous" functions?

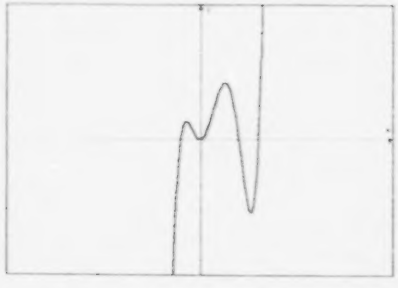
A



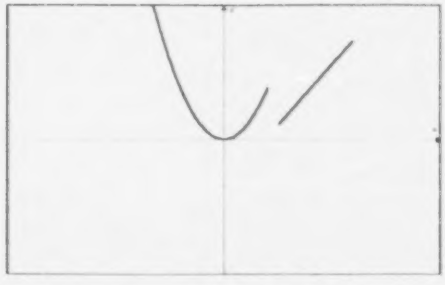
B



C



D



2. Read slides #4-11 and answer the following questions from slide #11:

Use the Leading Coefficient Test to determine the behavior of the graphs below when

$x \to \infty$ and $x \to -\infty$

a) $f(x) = x^3+2$ as $x \to \infty$ $f(x) \to$ _____

as $x \to -\infty$ $f(x) \to$ _____

b) $f(x) = 3x^4+2x-5$ as $x \to \infty$ $f(x) \to$ _____

as $x \to -\infty$ $f(x) \to$ _____

c) $f(x) = -2x^3-5x+1$ as $x \to \infty$ $f(x) \to$ _____

as $x \to -\infty$ $f(x) \to$ _____

d) $f(x) = 2x^5-x$ as $x \to \infty$ $f(x) \to$ _____

as $x \to -\infty$ $f(x) \to$ _____

e) $f(x) = -(x+2)(x-3)^2(x+4)^2$ as $x \to \infty$ $f(x) \to$ _____

as $x \to -\infty$ $f(x) \to$ _____

3. Read slides #12-17 and answer the following questions:

Find each real zero and its multiplicity for the polynomial functions. (Be sure to factor to factor each polynomial before you find its zeros)

a) $f(x) = x(x-3)(x+2)$ zeros multiplicity

b) $g(x) = x^5-5x^3+4x$ zeros multiplicity

c) $h(x) = 3x^2(x-1)(x-1)$ zeros multiplicity

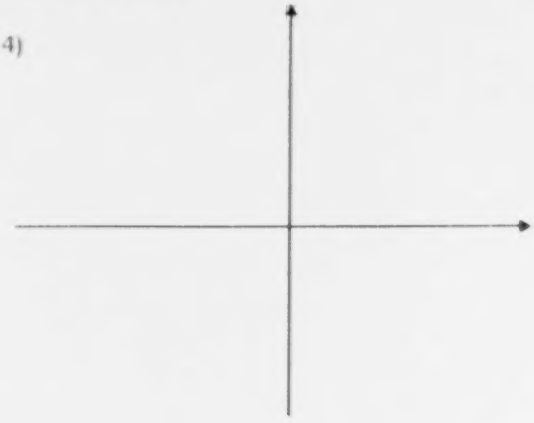
d) $p(x) = 7x^3-4x^2$ zeros multiplicity

e) $r(x) = -2x(x-2)^3(x+4)(x-5)$ zeros multiplicity

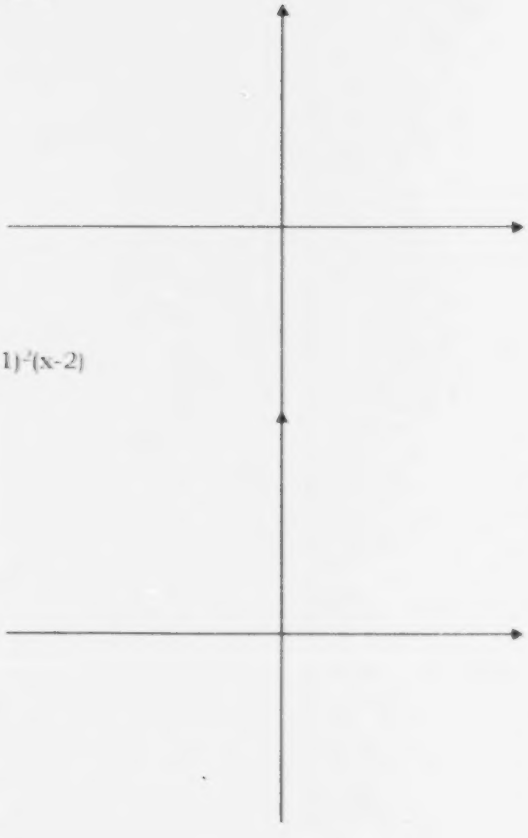
4. **Read slides #18-23 and answer the following questions:**

Use the sketching techniques described in the slides to sketch the following polynomial functions:

a) $f(x) = (x-3)(x+2)^3(x-4)$



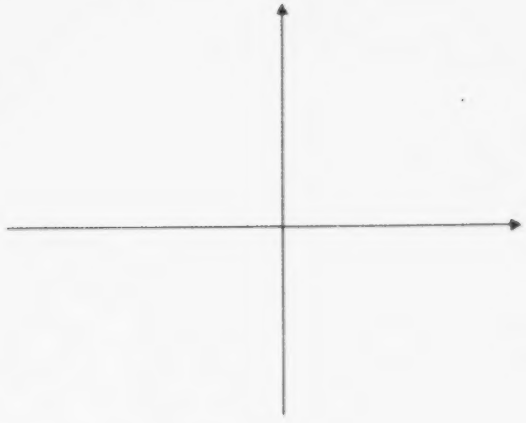
b) $f(x) = -4x(x+2)(x-5)^2(x-3)$



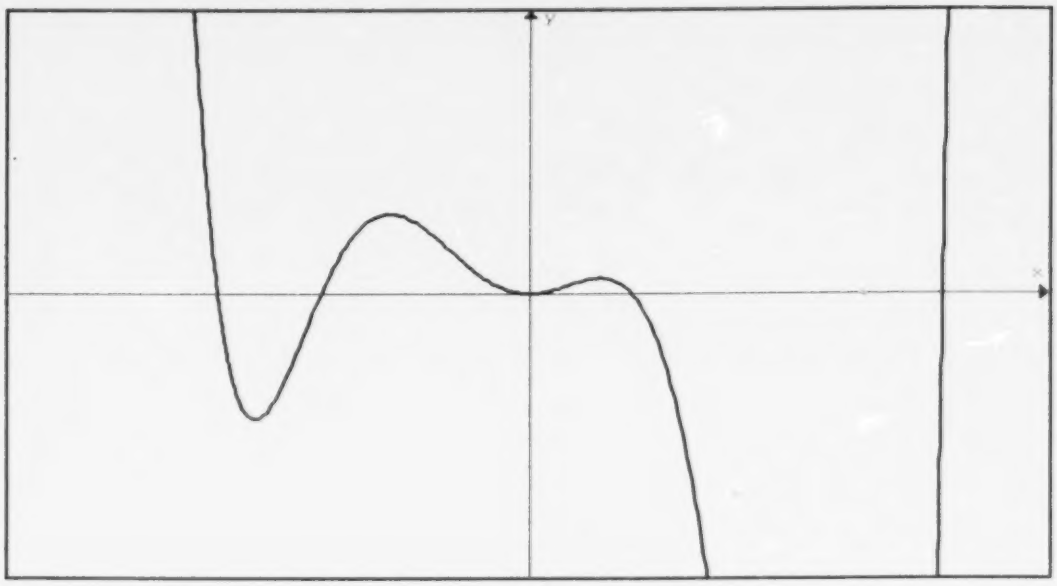
c) $f(x) = -0.5x^2(x+3)(x-1)^2(x-2)$

d) $h(x) = -2x^3(x-3)^2$

e) $f(x) = 0.5x^2(x-2)(x+3)$



A FEW MORE QUESTIONS FOR THOUGHT...



Shown above is the graph of a polynomial function.

a) **Is the degree of the polynomial even or odd?**

b) **Is the leading coefficient positive or negative?**

c) **Why is x^2 a factor of the polynomial?**

d) **What is the minimum degree of the polynomial?**

e) **Formulate three different polynomials whose graphs could look like the one shown above.**

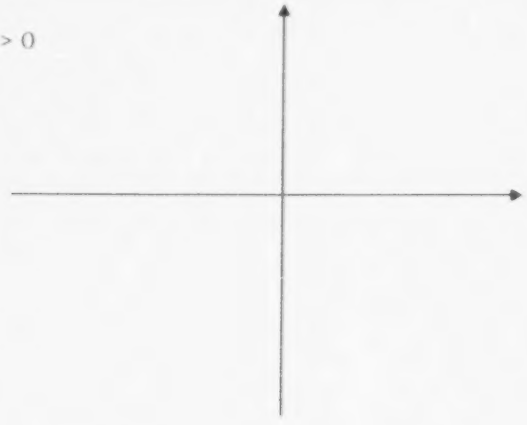
Compare yours to other group members. What do you see?

PART II

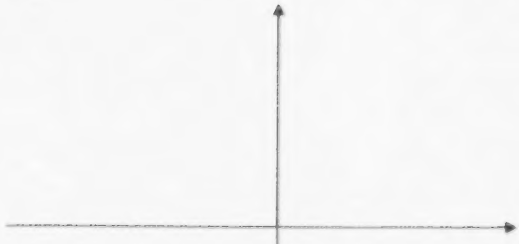
Solving Factored Inequalities in One Variable

1. Read slides #24-30 and answer the following questions:

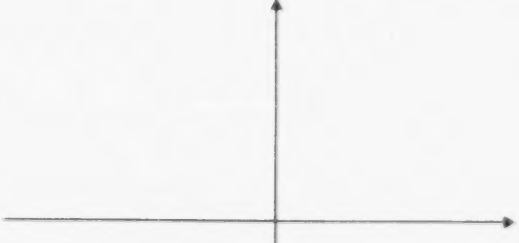
a) Solve: $-x(x-3)(x+2) > 0$



b) Solve: $x^3-10x^1+9x < 0$



c) Solve: $-3x^2(x-1)(x-1) > 0$



d) Solve: $x^2(x-3)^2(x+1)(x+2) < 0$

e) Solve: $5x^4-2x^3 > 0$

2. Answer question #1 again if the inequalities contain either $\leq$ or $\geq$ in the problem:

a) Solve: $-x(x-3)(x+2) \geq 0$ answer: _____

b) Solve: $x^5-10x^4 + 9x \leq 0$ answer: _____

c) Solve: $-3x^2(x-1)(x-1) \geq 0$ answer: _____

d) Solve: $x^2(x-3)^2(x+1)(x+2) \leq 0$ answer: _____

e) Solve: $5x^4-2x^3 \geq 0$ answer: _____

[redacted]

Vicki H. Norwich
Jacci W. White
Brevard Community College
Melbourne, FL 32937

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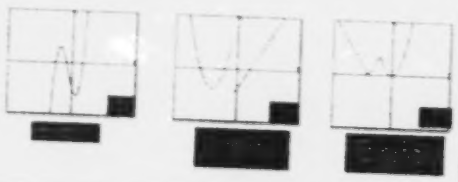
Graphing Factored Polynomials of Degree n > 1

Recall the basic graphs for polynomial functions of degrees 0, 1, and 2.

[redacted]

The graph of a polynomial function is a smooth curve. (We say **continuous** and you will study this concept in great detail in calculus). This means that the graph of a polynomial function has no breaks or sharp turns.

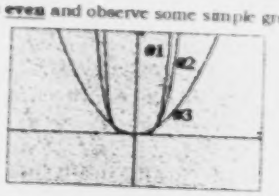
Consider the graphs below:



Let's consider the graph of $f(x) = x^n$ and focus on polynomial functions with degree > 1

Suppose **n** is **even** and observe some simple graphs:

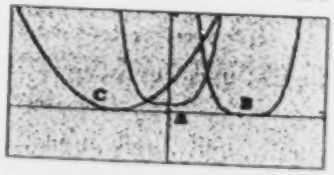
#1. $f(x) = x^2$
#2. $f(x) = x^4$
#3. $f(x) = x^6$



Notice that if n is even, the graph of x^n touches the x-axis at the x-intercept. Also notice that when **n** is even, the graph is similar to the graph of $f(x) = x^2$

Here are some more polynomial functions so that **n** is **even**.

A: $f(x) = x^4 + 1$ **B:** $g(x) = (x-3)^4$ **C:** $g(x) = (x+2)^2$



A: $f(x) = x^4 + 1$ The graph of f is an upward shift of $y=x^4$ by one unit.

B: $g(x) = (x-3)^4$ The graph of g is a right shift of $y=x^4$ by three units.

C: $h(x) = (x+2)^2$ The graph of h is a left shift of $y=x^2$ by two units.

Since we are considering the graph of $f(x) = x^n$ let's consider **n** is **odd** and observe some simple graphs:

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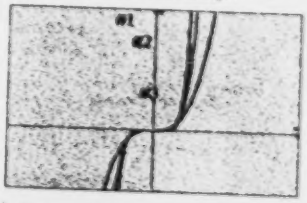

Notice that if n is odd, the graph of x^n crosses the x-axis at the x-intercept. Also notice that when **n** is odd, the graph is similar to the graph of $f(x) = x^3$

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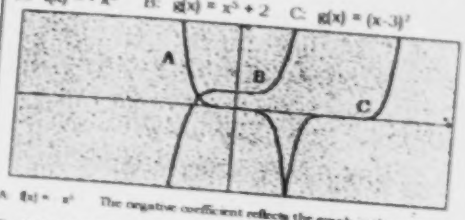
Here are some more polynomial functions so that n is odd.

A: $f(x) = -x^3$ B: $g(x) = x^3 + 2$ C: $g(x) = (x-3)^3$



A: $f(x) = -x^3$ The negative coefficient reflects the graph in the x axis.

B: $g(x) = x^3 + 2$ The graph of g is an upward shift of $y=x^3$ by two units.

C: $h(x) = (x-3)^3$ The graph of h is a right shift of $y=x^3$ by three units.

Consider the general form of a polynomial function:

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$

The algebra may seem complicated but suppose we factored out the leading term $a_n x^n$ so that we get:

$f(x) = a_n x^n \left(1 + \frac{a_{n-1}}{a_n}\frac{1}{x} + \frac{a_{n-2}}{a_n}\frac{1}{x^2} + \ldots + \frac{a_2}{a_n}\frac{1}{x^{n-2}} + \frac{a_1}{a_n}\frac{1}{x^{n-1}} + \frac{a_0}{a_n}\frac{1}{x^n}\right)$

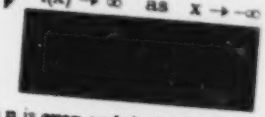
$= a_n x^n \left(1 + \frac{a_{n-1}}{a_n}\frac{1}{x} + \frac{a_{n-2}}{a_n}\frac{1}{x^2} + \ldots + \frac{a_2}{a_n}\frac{1}{x^{n-2}} + \frac{a_1}{a_n}\frac{1}{x^{n-1}} + \frac{a_0}{a_n}\frac{1}{x^n}\right)$

Each term that contains x in the parenthesis will be very small numbers when $|x|$ gets large. The term that dominates this polynomial will be the term $a_n x^n$, or the leading coefficient.

1) When **n** is **even** and the leading coefficient $a_n > 0$

$f(x) \to \infty$ as $x \to \infty$

$f(x) \to \infty$ as $x \to -\infty$



2) When **n** is **even** and the leading coefficient $a_n < 0$

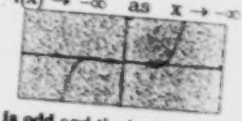
$f(x) \to -\infty$ as $x \to \infty$

$f(x) \to -\infty$ as $x \to -\infty$

3) When **n is odd** and the leading coefficient $a_n > 0$

$f(x) \to \infty$ as $x \to \infty$

$f(x) \to -\infty$ as $x \to -\infty$



4) When **n is odd** and the leading coefficient $a_n < 0$

$f(x) \to -\infty$ as $x \to \infty$

$f(x) \to \infty$ as $x \to -\infty$

NOW YOU TRY IT!!!

Use the Leading Coefficient Test to determine the behavior of the graphs below when

a) $f(x) = x^3+2$

b) $f(x) = -3x^4+2x-5$

c) $f(x) = -2x^3-5x+1$

d) $f(x) = 2x^6-x$

e) $f(x) = -(x+2)(x-3)^2(x+4)^2$

REAL ZEROS OF POLYNOMIAL FUNCTIONS

Remember that a real zero of a function f is a number c for which f(c) = 0

example: $f(x) = x^2 + x - 12$
$= (x-3)(x+4)$

3 is a zero of f because
$f(3) = (3-3)(3+4)$
$= (0)(4)$
$= 0$

-4 is a zero of f because
$f(-4) = (-4-3)(-4+4)$
$= (-7)(0)$
$= 0$

Find the zeros for the following functions:

a) $f(x) = x^2 - 4x$ zeros: _____

b) $f(x) = (2x-3)(x+1)$ zeros: _____

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REAL ZEROS OF POLYNOMIAL FUNCTIONS

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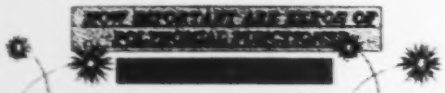
If *f* is a polynomial function and c is a real zero of *f*, then the statements below are equivalent:

1. **x = c is a zero of the function f.**
2. **x = c is a solution of the equation f(x) = 0.**
3. **(x-3) is a factor of the polynomial function f.**
4. **(c,0) is an x-intercept of the graph of *f*.**



We can often use information about the zeros of a function to help sketch its graph. We can also use information about the graph of a function to help find its zeros.

SEARCHING FOR ZEROS...

14

Do the following problems. Remember to factor each polynomial completely.

1) $f(x) = x(x-3)(x+2)$

2) $g(x) = x^5 - 5x^3 + 4x$

3) $h(x) = 3x^2(x-1)(x-1)$

4) $p(x) = (x-3)^2(x+1)(x+4)$

5) $r(x) = 5x^4 - 2x^3$

Do these problems on your worksheet !

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In some examples you should have seen real zeros that we call repeated zeros.

$h(x) = 3x^2(x-1)(x-1)$

zero is a repeated zero

and

one is a repeated zero

$p(x) = (x-3)^2(x+1)(x+4)$

three is a repeated zero

$r(x) = x^3(5x-2)$

zero is a repeated zero

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In general, we say that a factor $(x-c)^k$ will give us a repeated zero x = c of multiplicity k.

$h(x) = 3x^2(x-1)(x-1)$

zero is a repeated zero

and

one is a repeated zero

$p(x) = (x-3)^2(x+1)(x+4)$

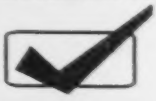
three is a repeated zero

$r(x) = x^3(5x-2)$

zero is a repeated zero

17

Suppose the polynomial function f has a factor $(x-c)^k$. That means that c is a zero of multiplicity k.



IMPORTANT !!!!

If k is odd, the graph will cross the x-axis at x = c and the "behavior" of the graph at c will be very much like the graph of $f(x)=(x-c)^k$ as it crosses the point (c, 0).

If k is even, the graph will touch (but it will not cross) the x-axis at x = c and the "behavior" of the graph at c will be very much like the graph of $f(x) = (x-c)^k$ as it touches (c, 0).

LET'S PUT ALL THIS TOGETHER NOW AND SEE HOW EASY IT IS TO SKETCH FACTORED POLYNOMIALS

18

Sketch $f(x) = (x-1)^2(x+2)$

leading term: x^3

zeros: 1 - multiplicity 2

-2 - multiplicity 1

y-intercept: $f(0) = (0-1)^2(0+2)$

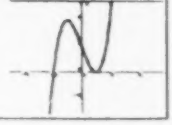
$= 2$

Since the leading coefficient is positive and the degree is odd, the function should finish in Quadrant III and Quadrant I

Let's start in Quadrant I
Graph must touch point (1,0)
(behaves like a parabola)

Graph continues to point (0,2)

Graph at zero point curves back and crosses (-2,0) like a linear function



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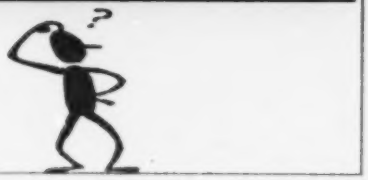

20

Sketch:

$f(x) = -\frac{1}{3}x^3(x+2)^2(x-3)$

leading term: $-\frac{1}{3}x^6$

zeros: 0- multiplicity 3

-2- multiplicity 2

3- multiplicity 1

y-intercept: 0

Since the leading coefficient is negative and the degree is even, the function should end up in Quadrant III and IV.

Let's start in Quadrant III for you can start in Q IV

Graph must touch (-2,0) like a parabola

Graph must cross (0,0) like a cubic function, and

It must cross (3,0) like a linear function so the sketch should look similar to the following...

21

Sketch:

$f(x) = 2x^2(x-2)(x+1)$

leading term: $2x^4$

zeros: 0- multiplicity 2

2- multiplicity 1

-1- multiplicity 1

y-intercept: 0

Since the leading coefficient is positive and the degree is even, the function should end up in Quadrant II and I.

Let's start in Quadrant II for you can start in Q I

Graph must touch (-1,0) and cross like a linear function

Graph must touch (0,0) like a parabola

Graph must cross (2,0) like a linear function so the sketch should look similar to the following...

22

SKETCH THE FOLLOWING GRAPHS ON YOUR WORKSHEET.

1) $f(x) = (x-3)(x+2)^3(x-4)$

2) $f(x) = -4x(x+2)(x-5)^2(x-3)$

3) $f(x) = -\frac{1}{2}x^2(x+3)(x-1)^2(x-2)$

4) $h(x) = -2x^3(x-3)^2$

5) $f(x) = \frac{1}{2}x^2(x-2)(x+3)$

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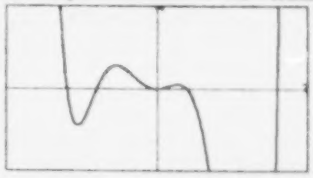
ANSWER THE QUESTIONS BELOW IN YOUR WORKSHEET...



Shown above is the graph of a polynomial function.

a) Is the degree of the polynomial even or odd?

b) Is the leading coefficient positive or negative?

c) Why is x^2 a factor of the polynomial?

d) What is the minimum degree of the polynomial?

e) Formulate three different polynomials whose graphs could look like the one shown above. Compare yours to other group members. What do you see?

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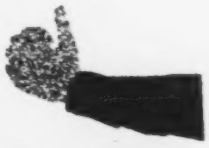
END OF PART I



DO NOT BEGIN PART II OF THIS EXERCISE UNTIL YOU HAVE FINISHED YOUR WORKSHEET FOR PART I.

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26

SOLVING FACTORED INEQUALITIES IN ONE VARIABLE

Let's look at a method for solving inequalities such as

$$(x - 3)(x + 2)(x - 1) > 0$$

or

$$x(x + 2)^2(x + 1)^2(x - 2) < 0$$

Let's use our new graphing techniques to solve these inequalities...

Solve the inequality $x(x+2)^2(x+1)^2(x-2) < 0$

28

Suppose we think about it like this...

Let $f(x) = x(x+2)^2(x+1)^2(x-2)$

Sketch:

Since the leading coefficient is positive and the degree is odd, the function should end up in Quadrant III and I

leading term: x^7

zeros: 0- multiplicity 1
-2- multiplicity 2
-1- multiplicity 3
2- multiplicity 1

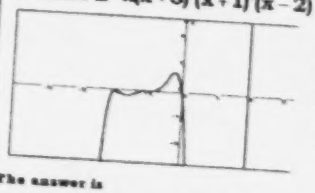
y-intercept: 0

Notice how the graph touches the point (-2,0) like a parabola, crosses the point (-1,0) like a cubic function, crosses the point (0,0) and the point (2,0) like a linear function

---

So where is $x(x+3)^2(x+1)^2(x-2) < 0$

27



The expression is less than zero where the graph of the function is below the x-axis, right?

The answer is

$-\infty < x < -2$ or $-2 < x < -1$ or $0 < x < 2$

In interval notation, the answer would be:

$(-\infty,-2) \cup (-2,-1) \cup (0,2)$

Solve the inequality $-0.5(x-3)(x-1)^2(x+2)^2 > 0$

29

Suppose we think about it like this...

Let $f(x) = -0.5(x-3)(x-1)^2(x+2)^2$

Sketch:

Since the leading coefficient is negative and the degree is even, the function should end up in Quadrant III and IV

leading term: $-0.5x^6$

zeros: 3- multiplicity 1
1-multiplicity 3
-2- multiplicity 2

y-intercept: $f(0) = -6$

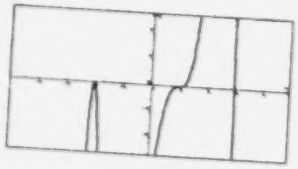
Notice how the graph touches the point (-2,0) like a parabola, crosses the point (1,0) like a cubic function, and crosses the point (0,0) (3,0) like a linear function

---

So where is $-0.5(x-3)(x-1)^2(x+2)^2 > 0$

29

The expression is greater than zero where the graph of the function is above the x-axis, right?



The answer is

$1 < x < 3$

In interval notation, the answer would be the interval: $(1,3)$

SOLVING MORE INEQUALITIES...

30

Use the sketching techniques from Part I to solve the following inequalities. Remember to factor each polynomial expression completely.

1) $-x(x-3)(x+2) > 0$

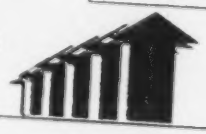
2) $x^5 - 10x^3 + 9x < 0$

3) $-3x^2(x+1)(x-1) > 0$

4) $x^2(x-3)^2(x+1)(x+2) < 0$

5) $8x^4 - 2x^2 > 0$

Do these problems on your worksheet I



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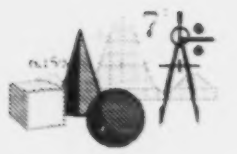


EXERCISE #8

Review of Quadratic Functions and Application Problems Using Quadratic Functions

Software:
Microsoft PowerPoint
Miscrosoft Excel

EXERCISE #8

Review of Quadratic Functions
and
Application Problems Using Quadratic Functions

Date: _____ **Group Members:**

_____

_____

Software Required:
PowerPoint
Microsoft Excel (optional)

_____

_____

In Part I, the student will review the standard equation $f(x) = ax^2 + bx + c$ of a quadratic function and the graph of a quadratic function. The student will identify the vertex and whether it is the lowest or highest point (minimum or maximum point) on the graph when the function is written in the form $f(x) = a(x-h)^2 + k$. The student will review how to complete the square to rewrite the parabola in the form $f(x) = a(x-h)^2 + k$. The student will utilize this information to solve application problems in Part II.

PART I
Quadratic Functions

1. Open the PowerPoint file titled "**QuadFun**" and read slides #1-3 to answer the following questions from slide #4.

   Identify the vertex of each parabola that appears below in standard form. State whether the vertex is the highest or lowest point on the parabola?

   a) $f(x) = (x-2)^2 + 3$ vertex _____ highest or lowest point? _____

   b) $f(x) = -3x^2+1$ vertex _____ highest or lowest point? _____

   c) $f(x) = -2(x+3)^2 - 1$ vertex _____ highest or lowest point? _____

   d) $f(x) = -4(x-5)^2$ vertex _____ highest or lowest point? _____

   e) $f(x) = (x+2)^2 + 5$ vertex _____ highest or lowest point? _____

   f) $f(x) = 3(x-6)^2 - 14$ vertex _____ highest or lowest point? _____

2. Read slides #5-7 and answer the following questions from slide #8:

- Complete the square to rewrite the function in the form $f(x)= a(x-h)^2 + k$
- identify the vertex of each parabola
- state whether it is the lowest or highest point of the graph.
- identify the maximum or minimum value of the function.

a) $f(x) = x^2 - 12x + 1$

new equation: _____

vertex _____ highest or lowest point? _____

maximum value _____

b) $g(x) = 4x^2 - 24x - 3$

new equation: _____

vertex _____ highest or lowest point? _____

maximum value _____

c) $f(x) = -2x^2 - 8x + 5$

new equation: _____

vertex _____ highest or lowest point? _____

maximum value _____

d) $g(x) = -3x^2 + 30x - 4$

new equation: _____

vertex _____ highest or lowest point? _____

maximum value _____

3. Read slides #9-11 and answer the following questions from slide #10:

Use the information that the vertex of the parabola $f(x) = ax^2 + bx + c \quad (a \neq 0)$ can also be found by using the formula $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a} \right) \right)$ to verify that your answers in #2 are correct. **Show your work.**

a) $f(x) = x^2 - 12x + 1$ vertex _____

a = ____ b = _____

b) $g(x) = 4x^2 - 24x - 3$ vertex _____

a = ____ b = _____

c) $f(x) = -2x^2 - 8x + 5$ vertex _____

a = ____ b = _____

d) $g(x) = -3x^2 + 30x - 4$ vertex _____

a = ____ b = _____

PART II

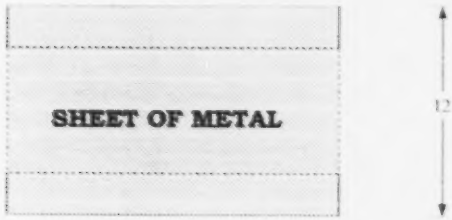
Application Problems Using Quadratic Functions

1. Read slides #12-25 and work Problem #1 as shown in the lesson.

PROBLEM #1

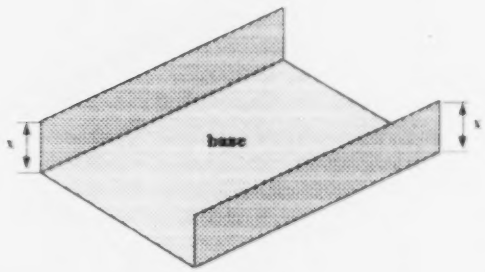
A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?

Step #1



Step #2

Step #3



The width of the base is x inches
The length of the cross-section rectangle is (12 - 2x) inches
Recall: The area of a rectangle is given by Area = (length)(base)

Remember, we want to determine the number of inches we should turn up to give the gutter its greatest capacity. That means the capacity will be greatest when the cross-sectional area of the rectangle with sides of lengths x and 12-2x has its greatest value.

Let f(x) denote this area: $f(x) = x(12-2x)$

Finish problem by finding the vertex of the quadratic function now. **SHOW WORK**.

Answer: _____

2. Read slide #26 to begin your work on Problem #2 as shown in the lesson.

PROBLEM #2

An apartment rental company has 2300 units available, and 800 are currently rented at an average of $450/month. A market survey indicates that each $15 decrease in average monthly rent will result in 50 new tenants.

Let x represent the number of $15 decreases in monthly rent. (For example, if x = 2, the rent is $420).

A) Write expressions that represent
   (a) the resulting rent per unit and
   (b) the resulting number of tenants.

B) Let R represent the total rental income and x represent the number of $15 decreases in monthly rent. Using what you found earlier, find an algebraic expression of R as a function of x.

C) Find the rent that will yield the rental company the maximum monthly income. (What is the domain of this function for this problem?)

SOLUTION:

A) Read slides #27-30 carefully to write expressions that represent the
   (a) the resulting rent per unit and
   (b) the resulting number of tenants (units)

Let's construct a spread sheet that will help us think about this problem. You might use the spreadsheet below from the lesson to help you determine the formulas that would go in each cell in the missing rows...

# of Decreases x	Avg Rent/ Month	Total Units Rented	Total Rental Revenue
0	450	800	360000
1	435	850	369750
2	420	900	378000
3			
4			
5			

Rent?
0 decrease? Rent = $450 - (0)15 = $450
1 decrease? Rent = $450 - (1)15 = $435
2 decreases? Rent = $450 - 2(15) = $420
3 decreases? Rent = $450 - 3(15) = $405
x decreases? Rent = $450 - x(15)

Units Rented?
0 decreases 800 + (0)50 = 800 units
1 decrease 800 + (1) 50 = 850 units
2 decreases 800 + (2) 50 = 900 units
3 decreases 800 + (3) 50 = 950 units
x decreases 800 + (x) 50

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B) Read slides #31-33 carefully to write an algebraic expression of R as a funtion of x.

Total Rental Revenue?

0 decreases	800 * $450	[800+ 0(50)] [450- 0(15)]
1 decrease	850 * 435	[800+ 1(50)] [450- 1(15)]
2 decreases	900 * 420	[800+ 2(50)] [450- 2(15)]
3 decreases	950 * 405	[800+ 3(50)] [450- 3(15)]
x decreases		[800+ x(50)] [450- x(15)]

$R(x) = (800+50x)(450-15x)$

Notice that our Revenue Income R is a quadratic function...

If we simplify the equation, we get

$$R(x) = 360000 - 12000x + 22500x - 750x^2$$

$$= -750x^2 + 10500x + 360000$$

When will we get the maximum value for R(x) ?

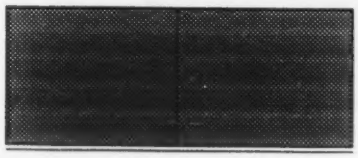
Compare it to your spreadsheet to see if the spreadsheet agrees?

SOLVING MORE PROBLEMS

You have been working problems that are sometimes referred to as optimization problems in mathematics. Do the following problems with your group members for additional practice.

PROBLEM #3

A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). What dimensions will produce a maximum enclosed area? **Show all work.**



PROBLEM #4 (Create a spreadsheet to help you solve this problem. See problem #2 and the sample spreadsheet below as an example) Show all work.

The owner of a 60-unit motel, by checking records of occupancy, knows that when the room rate is \$50/day, all units are occupied. For every increase of x dollars in the daily rate, x units are left vacant. Each unit occupied costs \$10 per day to service and maintain. Let P denote the total daily profit.

a) Determine a formula in one variable for P(x).
b) What is the maximum profit according to your spreadsheet?
c) Find the maximum value using algebraic methods and compare your answer with your spreadsheet.

P(x) = _____

Maximum profit = _____

# of x dollars increases	# of rooms rented	Income per Room	Total Rental Income	Expense for Rooms Rented	P Total Profit
0	60	50	3000	600	2400
1	59	51	3009	590	2419
2	58	52	3016	580	2436
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					

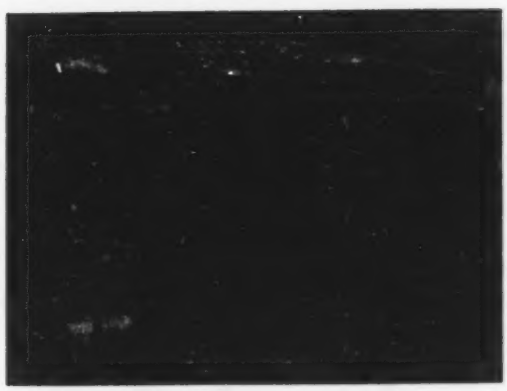
PROBLEM #5

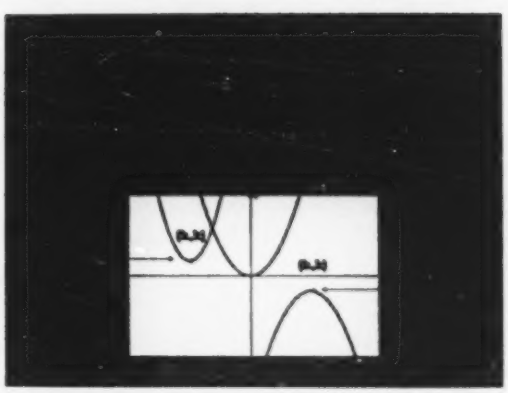
An object is thrown straight upward from a height of 6 feet with an initial velocity of 32 feet per second. The height at any time is given by

$s(t) = -16t^2 + 32t + 6$ where s(t) is measured in feet and t in seconds. Find the maximum height attained by the object before it beings falling to the ground. **Show work.**

Maximum height = _____

QUADRATIC FUNCTIONS and SOME APPLICATIONS





Identify the vertex of each parabola that appears below in standard form. State whether the vertex is the highest or lowest point on the parabola

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a) $f(x) = (x-2)^2 + 3$

b) $f(x) = -3x^2+1$

c) $f(x) = -2(x+3)^2 - 1$

d) $f(x) = -4(x-5)^2$

e) $f(x) = (x+2)^2 + 5$

f) $f(x) = 3(x-6)^2 - 14$

METHOD OF COMPLETING THE SQUARE 5

When given a parabola in the form $y=ax^2 + bx + c$ remember this method can be used to rewrite the parabola in the form $y = a(x-h)^2+k$:

EXAMPLE #1:

a = 1

$y = x^2 + 6x - 5$

$y = x^2 + 6x + 9 - 9 - 5$ add and subtract the square of one-half the coefficient of x

$y = (x+3)^2 - 9 - 5$

$y = (x+3)^2 - 14$ Parabola now in required form vertex: (-3, -5) and opens up

EXAMPLE #2: 6

a = 1

$y = 2x^2 + 20x - 1$

$y = 2(x^2 + 10x \quad) - 1$ factor the coefficient of x^2 from the first two terms

$y = 2(x^2 + 10x +25-25) - 1$ add and subtract the square of one-half the coefficient of x inside the parenthesis

$y = 2(x^2 + 10x +25) -50 - 1$ Rewrite expression so parenthesizes contains the perfect square trinomial

$y = 2(x+5)^2 - 51$ Parabola now in required form vertex: (-5, -51) and opens up

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EXAMPLE #3: 7

a= 1

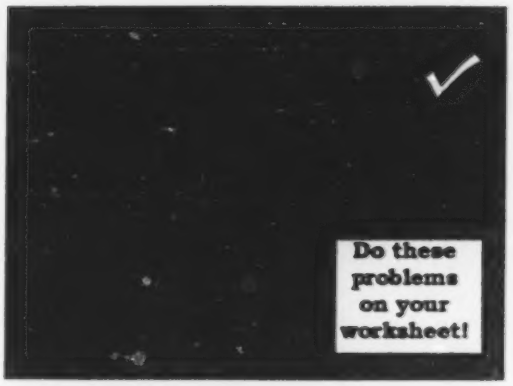
$y = -3x^2 + 24x + 7$

$y = -3(x^2 - 8x\quad) + 7$ — factor the coefficient of x^2 from the first two terms

$y = -3(x^2 - 8x + 16 - 16) + 7$ — add and subtract the square of one-half the coefficient of x inside the parenthesis

$y = -3(x^2 - 8x + 16) + 48 + 7$ — Rewrite expression so parentheses contains the perfect square trinomial

$y = -3(x-4)^2 + 55$ — Parabola now in required form vertex: (4, 55) and opens down



9

The statement can be proven by completing the square of a parabola in standard form. When you use the formula above, once the x-coordinate is found, you can calculate the y-coordinate by substituting -b/(2a) for x in the equation of the parabola.

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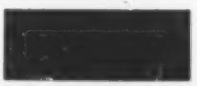

These are the same problems on your worksheet again. Use the formula for the vertex to verify that your answers are correct from completing the square.

1) $f(x) = x^2 - 12x + 1$
2) $g(x) = 4x^2 - 24x - 3$
3) $f(x) = -2x^2 - 8x + 5$
4) $g(x) = -3x^2 + 30x - 4$

Do these problems on your worksheet!

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END OF PART I

DO NOT BEGIN PART II OF THIS EXERCISE UNTIL YOU HAVE FINISHED YOUR WORKSHEET FOR PART I.

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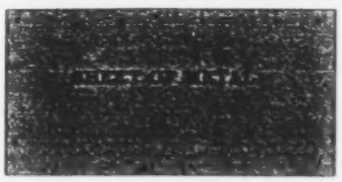
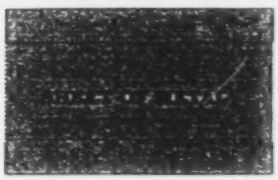
PROBLEM #1 13

A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?

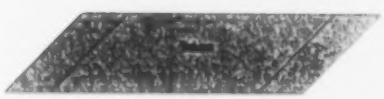


Basic Shape Used to Construct the Gutter 14



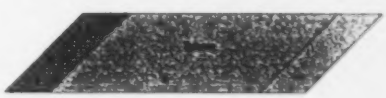
15

Visualize Construction of the Gutter



16

Select Left Side

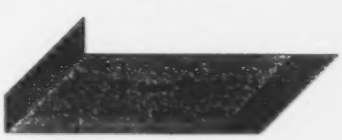


17

Bend Left Side Upward

18

Select Right Side



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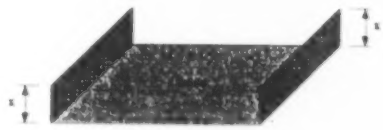
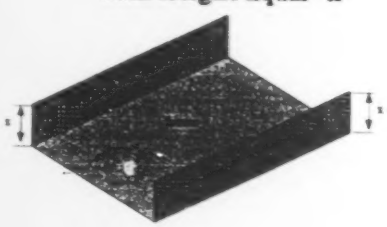
Bend Right Side Upward 19

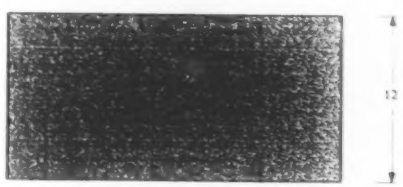
Finished Gutter With Height Equal "x" 20



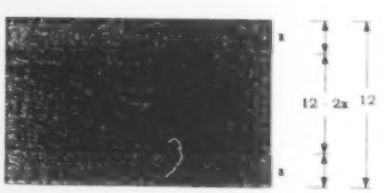
Side View of Finished Gutter With Height Equal "x" 21



Do it Algebraically Start with the Basic Rectangle 22



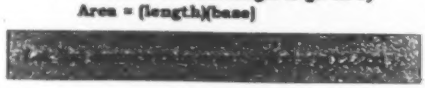
Identify the Inside Dimensions 23



Writing the Equation for the Problem 24

Remember, we want to determine the number of inches we should turn up to give the gutter its greatest capacity. That means the capacity will be greatest when the cross-sectional area of the rectangle with sides of lengths x and 12-2x has its greatest value.

- **The length of the cross-section rectangle is (12 - 2x) inches**
- **The width of the base is x inches**
- **The area of a rectangle is given by Area = (length)(base)**



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Solving the problem

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$f(x) = x(12-2x)$ Area of the cross section of the rectangle

$= 12x - 2x^2$ Distributive Property

$= -2x^2 + 12x$ Write right side in standard form

Since this is a quadratic function, $a = -2$, $b = 12$, and $c = 0$.

The graph of the function is a parabola that opens downward. The vertex of the parabola is at $x = -b/(2a) = (-12)/(-4)$ or $x = 3$. So the parabola has its maximum value at $x = 3$.

[redacted]

PROBLEM #2

26

An apartment rental company has 2300 units available, and 800 are currently rented at an average of $450/month. A market survey indicates that each $15 decrease in average monthly rent will result in 50 new tenants.

Let x represent the number of $15 decreases in monthly rent. (For example, if $x = 2$, the rent is $420).

1) Write expressions that represent the
   (a) the resulting rent per unit and
   (b) the resulting number of tenants.

2) Let R represent the total rental income and x represent the number of $15 decreases in monthly rent. Using what you found in #1, find an algebraic expression of R as a function of x.

3) Find the rent that will yield the rental company the maximum monthly income. (What is the domain of this function for this problem situation?)

Statement of Problem #2 again...

27

An apartment rental company has 2300 units available, and 800 are currently rented at an average of $450/month. A market survey indicates that each $15 decrease in average monthly rent will result in 50 new tenants.

Let's construct a spread sheet that will help us think about this problem.

# of Decreases x	Avg Rent/ Month	Total Units Rented	Total Rental Revenue
0	450	800	360000
1	435	850	369750
2	420	900	378000
3	405	950	384750
4	390	1000	390000
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

Where did these numbers come from in this spreadsheet?!

Let's go back and work out the formulas for the cells to finish the spreadsheet...

Statement of Problem #2 again...

28

An apartment rental company has 2300 units available, and 800 are currently rented at an average of $450/month. A market survey indicates that each $15 decrease in average monthly rent will result in 50 new tenants.

# of Decreases x	Avg Rent/ Month	Total Units Rented	Total Rental Revenue
0	450	800	360000
1	435	850	369750
2	420	900	378000
3	405	950	384750
4	390	1000	390000
5			
6			
7			
8			
9			
10			

Determine the formulas that would go in each cell from row 5 down...

Statement of Problem #2 again...

29

An apartment rental company has 2300 units available, and 800 are currently rented at an average of $450/month. A market survey indicates that each $15 decrease in average monthly rent will result in 50 new tenants.

1) **Write expressions that represent the**
   (a) the resulting rent per unit and

1 decrease? Rent = $450 - $15 = $435

2 decreases? Rent = $450 - 2(15) = $420

3 decreases? Rent = $450 - 3(15) = $405

x decreases? Rent = $450 - x(15)

(b) the resulting number of tenants.

30

0 decreases 800 + (0)50 = 800 units

1 decrease 800 + (1) 50 = 850 units

2 decreases 800 + (2) 50 = 900 units

3 decreases 800 + (3) 50 = 950 units

x decreases 800 + (x) 50 =

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2) Let R represent the total rental income and x represent the number of $15 decreases in monthly rent. Using what you found in #1, find an algebraic expression of R as a function of x.

R(0) = 800 * $450 [800+ 0(50)] [450- 0(15)]

R(1) = 850 * 435 [800+ 1(50)] [450- 1(15)]

R(2) = 900 * 420 [800+ 2(50)] [450- 2(15)]

R(3) = 950 * 405 [800+ 3(50)] [450- 3(15)]

R(x) = ? [800+ x(50)] [450- x(15)]

➡ **R(x) = (800+50x) (450-15x)**

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3) Find the rent that will yield the rental company the maximum monthly income. (What is the domain of this function for this problem?)

Now we want to maximize the total rental revenue for the company. From the spreadsheet, it looks like the revenue would be maximized with 7 decreases. Is the spreadsheet correct?

Also, the domain (x) for this function would be:

$0 \le x \le 30$

# of Decreases	Avg. Rent/ Month	Total Units Rented	Total Rental Revenue
0	$450	800	$360,000
1	$435	850	$369,750
2	$420	900	$378,000
3	$405	950	$384,750
4	$390	1000	$390,000
5	$375	1050	$393,750
6	$360	1100	$396,000
7	$345	1150	$396,750
8	$330	1200	$396,000
9	$315	1250	$393,750
10	$300	1300	$390,000
11	$285	1350	$384,750
12	$270	1400	$378,000
13	$255	1450	$369,750
14	$240	1500	$360,000
15	$225	1550	$348,750
16	$210	1600	$336,000
17	$195	1650	$321,750
18	$180	1700	$306,000
19	$165	1750	$288,750
20	$150	1800	$270,000
21	$135	1850	$249,750
22	$120	1900	$228,000
23	$105	1950	$204,750
24	$90	2000	$180,000
25	$75	2050	$153,750
26	$60	2100	$126,000
27	$45	2150	$96,750
28	$30	2200	$66,000
29	$15	2250	$33,750
30	$0	2300	$0

33

R(x) = (800+50x) (450-15x) !!!!

Notice that our Revenue Income R is a quadratic function...

If we simplify the equation, we get

$R(x) = 360000 - 12000x + 22500x - 750x^2$

$= -750x^2 + 10500x + 360000$

R(x) will be at its maximum value at the vertex or where $x = -b/(2a) = -(10500)/(-750) = 7$

Why are we NOT surprised?!! With 7 decreases, the revenue was at its highest in the spreadsheet. So the spreadsheet is correct.

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SOLVING MORE PROBLEMS

In mathematics, you have been working problems that are sometimes referred to as optimization problems. Do these problems in your worksheet.

PROBLEM #3

A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). What dimensions will produce a maximum enclosed area?

PROBLEM #4

The owner of a 60 unit motel, by checking records of occupancy, knows that when the room rate is $50/day, all units are occupied. For every increase of x dollars in the daily rate, x units are left vacant. Each unit occupied costs $10 per day to service and maintain. Let P denote the total daily profit. Create a spreadsheet for this problem.

a) Determine a formula in one variable for P(x).

b) What is the maximum profit according to your spreadsheet?

c) Algebraically find the maximum value of the function P.

d) Compare your responses.

PROBLEM #5

An object is thrown straight upward from a height of 6 feet with an initial velocity of 80 feet per second. The height at any time is given by

$s(t) = -16t^2 + 32t + 6$ where s(t) is measured in feet and t in seconds. Find the maximum height obtained by the object before it beings falling to the ground.

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EXERCISE #9

Shifting Graphs Including Horizontal, Vertical, and Combination Graph Shifts

Software: Derive

EXERCISE #9

Shifting Graphs

Date: _____

Group Members:

_____

_____

_____

_____

Software Required:
Derive

The student will explore connections between constants in equations and associated shifts in a basic graph. These shifts will include horizontal, vertical, and combinations of these shifts.

PART 1

Single Shifts

1. Use *Derive* to help you sketch the following functions.

$f(x) = x^2$

$f(x) = x^2 + 2$

$f(x) = x^2 + 5$

$f(x) = x^2 - 3$

$f(x) = x^2 + 9$

$f(x) = x^2 - 4$

2. What do you notice about these graphs? Are the shapes the same? How could you use the first graph to help you sketch the other equations?

3. Make a rule to generalize what is happening to the graph when you have

$$f(x) = x^2 + c$$

4. Does it matter if the constant is positive or negative? If so, how?

5. Do you think your rule will generalize to any function where y=f(x)+c?

6. Use *Derive* to help you sketch the following functions.

- $f(x) = \sqrt{x}$

$f(x) = \sqrt{x+1}$

$f(x) = \sqrt{x+8}$

$f(x) = \sqrt{x-5}$

$f(x) = \sqrt{x-3}$ $f(x) = \sqrt{x+9}$

7. What do you notice about these graphs? How are the graphs similar? How are they different?

8. Make a rule to generalize what is happening to the graph when you have

$$f(x) = \sqrt{x+c}$$

9. Does it matter if the constant is negative?

10. Do you think your rule will generalize to y=f(x+c)?

PART II

Graph Shift Combinations

1. Use your rules from Part I to predict what will happen when you sketch the graph for each of the following equations.

$f(x) = (x-2)^2 + 1$

$f(x) = (x+4)^2 - 8$

$f(x) = \sqrt{x-1} - 2$

$f(x) = \sqrt{x+5} - 6$

$f(x) = (x+4)^2 + 4$

$f(x) = \sqrt{x-7} + 1$

2. Do your rules hold true? If no, revise your rules.

3. Discuss your rules with another group and revise again if necessary.

Technology Review

by
Vicki Norwich
and
Jacci White

TECHNOLOGY COMPARISON

- ISETL
- DERIVE
- Geometer's Sketchpad
- TI-82, TI-83
- TI-85, TI-86
- TI-92
- CBL

ISETL

A Mathematical Programming Language

ISETL

- FREEWARE
- Available from the Internet, or West Publishing
- IBM DOS, Windows, or Mac Version available

ISETL

- Write programs to create functions.
- Write programs to graph functions.
- Write programs to evaluate functions.
- Write programs for all operations.

SAMPLE ISETL PROGRAM

- f:=func(x);
  - if x>0 then return x**2-1;
  - elseif x=0 then return 1;
  - else return x+1;
  - end;
  - end;

ISETL ADVANTAGES

- FREE
- EASY TO INSTALL
- NO ADDITIONAL PURCHASE NECESSARY

ISETL DISADVANTAGES

- Must learn the programming language.
- Operations must be programmed.
- Assistance is hard to find.
- Time consuming to use until it becomes familiar and a store of programs have been developed.

DERIVE

SYMBOLIC COMPUTER SYSTEM

DERIVE

- Must be purchased
- Can be ordered from local dealers
- Student version available

DERIVE

- Enter problems the way they appear in the text
- Evaluate functions
- Graph functions
- Perform all basic Calculus computations and operations
- Perform algebraic manipulations

SAMPLE DERIVE APPLICATIONS

- Evaluate a function at different values
- Graph any function in 2 or 3 dimension
- Solve equations

DERIVE ADVANTAGES

- Easy to use, no assistance necessary
- Can display the answer in different forms
- Fast
- Problems appear in actual form

DERIVE DISADVANTAGES

- COST
- Need a computer

GEOMETER'S SKETCHPAD

A Dynamic Geometry System

GEOMETER'S SKETCHPAD

- Must be purchased
- Can be ordered from local dealers
- Student version available

GEOMETER'S SKETCHPAD

- Sketch geometric figures including angles, bisectors, perpendicular lines, triangles, circles, and more
- Evaluate functions
- Graph functions
- Perform numeric computations

SAMPLE GEOMETER'S SKETCHPAD APPLICATIONS

- Sketch geometric figures
- Calculate angles, midpoints, lengths, and more
- Animate sketches to illustrate identities and theorems

GEOMETER'S SKETCHPAD ADVANTAGES

- Easy to use, directions are fairly straight forward
- Little background in Geometry is necessary to get started
- Fast
- Very useful as a discovery tool for visualizing new concepts

GEOMETER'S SKETCHPAD DISADVANTAGES

- COST
- Need a computer
- Designed for Geometry so not as useful in other mathematics courses

TI - 82

Graphing Calculator

TI-82

- Approximately $89.00
- Available from local dealers as well as office supply stores, and Target, Service Merchandise, etc...
- Designed for use up to College Algebra

TI-82

- Programmable
- Can printout on the computer using Graphlink
- Links to CBL
- Shares viewscreen with TI-83

SAMPLE TI-82 APPLICATIONS

- Evaluate functions
- Graph functions
- Perform basic Calculus operations and functions

TI-82 ADVANTAGES

- Cost is low for a graphing calculator
- Displays data in tabular form
- Menus are easy to use

TI - 82 DISADVANTAGES

- Problems must be entered using the correct order of operations
- Less memory space than more expensive models
- Limited number of advanced functions
- No algebraic manipulations

TI - 85

GRAPHING
CALCULATOR

TI - 85

- Approximately $99.00
- Available from local dealers as well as office supply stores, Target, Walmart, etc...
- Designed for use up to Calculus II

TI - 85

- Programmable
- Can printout on the computer using Graphlink
- Links to CBL
- Shares Viewscreen with TI-86

Sample TI-85 Applications

- Graph functions
- Perform many Calculus I and II operations
- Answers in decimal, fraction, or complex number form

TI - 85 ADVANTAGES

- More memory than the TI-82
- All commands can be found under the catalog key
- Many advanced engineering capabilities

TI - 85 DISADVANTAGES

- No table feature
- Slightly more expensive than the TI-82
- No algebraic manipulations

TI - 86

GRAPHING CALCULATOR

TI - 86

- Approximately $99.00
- Available from local dealers as well as office supply stores, Target, Walmart, etc...
- Designed for use up to Calculus II and Defferential Equations

TI - 86

- Programmable
- Can printout on the computer using Graphlink
- Links to CBL
- Shares Viewscreen with TI-85

Sample TI-86 Applications

- Graph functions
- Perform many Calculus I and II operations
- Answers in decimal, fraction, or complex number form
- Performs many procedures from Differential Equations

TI - 86 ADVANTAGES

- More memory than the TI-85
- All commands can be found under the catalog key
- Many advanced engineering and Differential Equations capabilities
- Contains the table feature to display data

TI - 86 DISADVANTAGES

- Slightly more expensive than the TI-82
- No algebraic manipulations
- No 3-D graphing

TI - 92

GRAPHING
CALCULATOR

TI - 92

- Approximately $190.00
- Available from most dealers and a few office supply stores
- Designed for use up to Calculus III and advanced mathematics courses

TI - 92

- Programmable
- Has Derive built in
- Has Cabri Geometry capabilities
- Can printout on the computer

Sample TI-92 Applications

- Algebraic manipulations
- Graphing including 3-space
- Does all Calculus I,II, and III applications
- Geometry

TI - 92 ADVANTAGES

- Algebraic manipulations
- Derive command structure
- 3 - D graphing
- Geometry applications

TI - 92 DISADVANTAGES

- Cost is higher than other calculators

TI - 83

GRAPHING CALCULATOR

TI - 83

- Approximately $100.00
- Available in the same locations as other Texas Instruments calculators
- Designed for use up to College Algebra and Statistics

TI - 83

- Split screen
- Programmable
- Can be used with Graphlink

Sample TI - 83 Applications

- Graph split with table feature
- Spreadsheet capabilities
- Statistics
- Business

TI - 83 ADVANTAGES

- Designied with more advanced statistical and business menu options.

TI - 83 DISADVANTAGES

- No new Calculus features
- No algebraic manipulations
- Slightly more expensive than the TI-82

CBL

Calculator
Based
Laboratory

CBL

- Approximately $200.00
- Available from local Dealers and Vernier Software

Sample CBL Applications

- Motion
- Temperature
- Light
- Many more

CBL ADVANTAGES

- Visualize real applications
- Experiment with new ideas
- Portable

CBL DISADVANTAGES

- Limited use for calculations
- Applications are not obvious to students
- Designed for classroom demonstration or group exploration